PACIFIC JOURNAL OF MATHEMATICS
Vol. 27, No. 1, 1968

A TAUBERIAN RELATION BETWEEN THE BOREL AND THE LOTOTSKY TRANSFORMS OF SERIES

SORAYA SHERIF

This paper is concerned with the equiconvergence of the Lototsky transform and the Borel (exponential) transform for a class of series satisfying the Tauberian condition $a_n = o(1)$.

If $s_n = a_0 + a_1 + \cdots + a_n$, the Borel (exponential) transform $f(x)$ of s_n is usually defined by

$$e^{-x} \sum_{n=0}^{\infty} s_n \frac{x^n}{n!} .$$

Writing $s_n = a_1 + a_2 + \cdots + a_n$, the Lototsky transform σ_n of s_n introduced by A. V. Lototsky [8] is defined by

$$\sigma_n = \frac{1}{n!} \sum_{k=1}^{n} p_{n,k} s_k \ , \tag{1.1}$$

where $p_{n,k}$ is the coefficient of x^k in

$$p_n(x) = x(x+1)(x+2) \cdots (x+n-1) \ , \qquad (n = 1, 2, \cdots) .$$

Thus it is usual in considering Lototsky summability to take the first term of the series as a_1, and in considering Borel summability[1] to take it as a_0. In order to compare the methods without changing the customary notation we will therefore apply the Borel methods to the series $0 + a_1 + a_2 + \cdots$ and apply the Lototsky method to the series $a_1 + a_2 + \cdots$. We recall (Hardy [5] pp. 182-3) that the Borel summability of $a_1 + a_2 + \cdots$ implies the Borel summability $0 + a + a + \cdots$, but not conversely. The two methods are equivalent if (and only if) $a_n \to 0(B)$; this is true in particular if

$$a_n = o(1) \ , \tag{1.2}$$

and thus for the series considered in this paper.

Lototsky's transform is essentially a special case of a class of transformations introduced by J. Karamata [7]. It is the (f, d_n) transform defined by G. Smith [11], when $f(z) = z$, $d_n = n$, and the $[F, d_n]$ transform defined by A. Jakimorski [6], when $d_n = n - 1$ and $n \geqq 1$. It is also the σ^α method of summability introduced by Vučković [12], when $\alpha = 1$.

Numerous properties of this Lototsky transform and its relation

[1] "Borel summability" is throughout taken to refer to Borel's exponential method.