PACIFIC JOURNAL OF MATHEMATICS
Vol. 27, No. 1, 1968

ON A CHARACTERIZATION OF INFINITE COMPLEX MATRICES MAPPING THE SPACE OF ANALYTIC SEQUENCES INTO ITSELF

LOUISE A. RAPHAEL

Let S be the space of all complex sequences. An element $u = \{u_n\}_{n=0}^{\infty}$ of S is called analytic if for some constant $M > 0$, $|u_n| \leqq M^{n+1}$ for $n = 0, 1, 2, \cdots$. By A denote the space of all analytic sequences. Clearly A is the space of all complex functions analytic at zero. I. Heller has proved

Theorem 1. The transformation $y_n = \sum_{m=0}^{\infty} c_{nm}u_m$ maps A into A if and only if for every $p > 0$ there exists a $q > 0$ and a constant $M > 0$ such that $|c_{nm}| \leqq Mp^m/q^n$ for $m, n = 0, 1, 2, \cdots$; and also if and only if the function G of two complex variables (i.e., in $E \times E$, where E is the complex plane) respresented by the double power series $G(z, y) = \sum_{m,n=0}^{\infty} c_{nm}z^m y^n$ be regular on $E \times 0$.

The present paper provides an alternative proof for the theorem in order to give insight into the structure of A as a countable union of BK spaces, that is, Banach spaces with coutinuous coordinates.

Let $q > 0$ be fixed and $A_q = \{u \in S \mid \sup_n |q^n u_n| = \|u\|_q < \infty, n = 0, 1, 2, \cdots\}$.

THEOREM 2. (1) *$A = \bigcup_{n=0}^{\infty} A_{q_n}$ where $q_n \downarrow 0$, and*
(2) *for any $q > 0$, $(A_q, \|u\|_q)$ is a BK space.*

Proof. (1) A complex sequence $u = \{u_n\}_{n=0}^{\infty}$ is analytic if and only if the $\sup_n |q^n u_n| \leqq M$ for some $q > 0$, some constant $M > 0$ and $n = 0, 1, 2, \cdots$. It now follows that $A = \bigcup_{0<q<\infty} A_q$. The proof is completed by a set theoretic argument showing that $\bigcup_{0<q<\infty} A_q = \bigcup_{n=0}^{\infty} A_{q_n}$ after observing that if $0 < r < s$, then $A_s \subset A_r$.

(2) It suffices to observe that $(A_q, \|u\|_q)$ is isometrically isomorphic with the Banach space of all bounded complex sequences

$$(m) = \{u \in S \mid \|u\|_{(m)} = \sup_n |u_n|\} .$$

The operator E_q from A_q into (m) establishing this isomorphism is defined by $E_q: \{u_n\}_{n=0}^{\infty} \rightarrow \{q^n u_n\}_{n=0}^{\infty}$. Finally for each n, $|u_n| \leqq \|u\|_q/q^n$. Thus the coordinate functional $P_n(u) = u_n$ is continuous, being a linear operator on A_q. This proves that the space $(A_q, \|u\|_q)$ is a BK space.

By a mapping C of a sequence space X into a sequence space Y generated by an infinite complex matrix (c_{nm}) $m, n = 0, 1, 2, \cdots$ is