PACIFIC JOURNAL OF MATHEMATICS
Vol. 27, No. 1, 1968

ON SUPPORTS OF REGULAR BOREL MEASURES

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The existence of a regular Borel measure whose support is a given compact Hausdorff space X imposes definite structures on $X, C(X)$, and $C(X)^*$. In this paper a necessary and sufficient condition is given to insure that X is the support of a regular Borel measure. This involves the intersection number of a collection of open sets in X. Measures which vanish on a sigma ideal of a sigma field of subsets of X which contains a basis for the topology of X are also considered. In particular, for a certain class of compact Hausdorff spacs X, necessary and sufficient conditions are given to insure the existence of a nonatomic regular Borel measure whose support is X. The final section of the paper is devoted to a study of normal measures; i.e., measures which vanish on meager Borel sets. Normal measures on X are shown to be related to normal measures on the projective resolution of X.

NOTATION AND TERMINOLOGY. Set theoretical and topological terminology is that of [12], the terminology of linear topological spaces is that of [14], and measure theory terminology follows [11]. All spaces considered are taken to be nonempty and all measures considered are finite. If X is a compact Hausdorff space, $C(X)$ denotes the space of continuous real-valued functions on X in the supremum norm, $C(X)^*$ denotes the space of all continuous linear functionals on $C(X)$, or, equivalently, the space of all signed regular Borel measures, and $B(X)$ denotes the space of all bounded real-valued functions on X in the supremum norm.

1. Intersection numbers.

The following definitions are motivated by the concept of an intersection number as given in [13]. Let X be a compact Hausdorff space and B be a Boolean algebra.

1.1. If $S = (f_1, \cdots, f_n)$ is a finite sequence in $B(X)$, $i(S) = (1/n) \| \sum_{i=1}^{n} f_i \|$. If $A \subseteq C(X)$, then $I(A) = \inf \{i(S) \colon S$ is a finite sequence in $A\}$.

1.2. If $S = (A_1, \cdots, A_n)$ is a finite sequence of subsets of X, $i(S) = \max \{(k/n) \colon$ there is a subsequence $(A_{i_1}, \cdots, A_{i_k})$ of S such that $\bigcap_{j=1}^{k} A_{i_j} \neq \varnothing\}$. If H is a collection of subsets of X, then $I(H) = \inf \{i(S) \colon S$ is a finite sequence in $H\}$.

1.3. If $S = (E_1, \cdots, E_n)$ is a finite sequence in B, then $i(S) =$