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CHARACTERIZING PRIMES IN SOME NONCOMMUTATIVE RINGS

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For a ring R with identity 1, a preprime is a nonempty subset T of R which is closed under the two binary operations, addition and multiplication, of R and with $-1 \notin T$. A prime of R is a preprime of R which is maximal with respect to set inclusion. A field K is locally finite if every member of K is a member of some finite subfield of K. For a finite dimensional vector space V over K let $\mathfrak{G} = \mathrm{Hom}_K(V, V)$ denote the full ring of linear transformations of V over K. Let W and L be subspaces of V with $W \subset L \subset V$ aud $W \neq L$. Let $T(L, W) = \{\alpha \in \mathfrak{G} / \alpha \cdot L \subset W\}$. Then $T(L, W)$ is a preprime of $\mathfrak{G}$. Let

$$\mathscr{T} = \{T(L, W)/W, L \text{ are subspaces of } V, W \subset L, W \neq L\}$$

We will show that the primes of $\mathfrak{G}$ are exactly those preprimes $T(L, W) \in \mathscr{T}$ with $\dim_K L = 1 + \dim_K W$.

There is also an associative monoid with zero element reminiscent of a value group for a valuation of a field. One actually finds that this monoid is independent of which prime is used to define it. However, this shows rather that while this concept yields an abelian group when the ring is commutative, it may not be the proper concept in noncommutative rings, see [1, Prop. 2.2].

A number field is a finite field extension of the field Q of rational numbers. In [1, Prop. 3.4, 3.5, 3.6], Harrison has shown that for a number field K, the primes are exactly the useful prime divisiors of algebraic number theory and all of them when K is a normal extension of Q. Since the definition of primes is made in arbitrary rings with 1, it is desirous to investigate the concept for nonfields. Commutative rings have been investigated considerably in [1] and [3]. A locally finite field has no primes but $\{0\}$ (see [1, Lemma 1.4]). Thus one would expect $\mathfrak{G} = \mathrm{Hom}_K(V, V)$ to be one of the simplest noncommutative rings to investigate.

All rings are assumed to have an identity. If A and B are subsets of a ring R and d a member of an R-module, dA denotes $\{da/A \in A\}$, Ad denotes $\{ad/a \in A)$, $A \cdot B$ denotes $\{ab/a \in A, b \in B\}$ and $-A$ denotes $\{-a/a \in A\}$. K will denote a locally finite field, V a finite dimensional vector space over K, and $\mathfrak{G} = \mathrm{Hom}_K(V, V)$ is the ring of all K-linear transformations of V.

1. P-productive. A prime P of a ring is called *finite* if $-1 \notin P$ *infinite* otherwise. P is finite if and only if $-P \subset P$. Since the