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ON COMMUTATIVE, NONPOTENT ARCHIMEDEAN SEMIGROUPS

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In this paper we will study commutative, archimedean, nonpotent (i.e., without an idempotent) semigroups, obtaining several results concerning finitely generated ones. The main theorem of this paper is the following: a finitely generated, commutative, archimedean, nonpotent semigroup is power joined. The main theorem is derived by considering the decomposition of the semigroup S into a union of disjoint semilattices; the congruence ρ_b, defined by $x\rho_b y$ if and only if there exist positive integers n and m such that $b^n x = b^m y$, determines the union, whereas congruence classes are semilattices under the partial order $\geqq_b$ defined by $x \geqq_b y$ if and only if $y = b^n x$ or $y = x$. The set of maximal elements relative to $\geqq_b$ generates S. The following is a crucial lemma in the proof of the main theorem: let S be a finitely generated, commutative, nonpotent, archimedean semigroup; then the set of maximal elements of S relative to $\geqq_b$ is a finite set.

Let S be a commutative, nonpotent, archimedean semigroup. We will define a congruence ρ on S and state several results concerning S/ρ and the congruence classes of S modulo ρ. The remarks and definitions which precede Definition 5 will be used in several instances; a complete discussion can be found in [5]. See [6] and [7] for an abstract of these results. Proofs of all other results in this paper are supplied.

DEFINITION 1. Let $b \in S$. The binary relation ρ_b on S is defined by $x\rho_b y$ if and only if there exist positive integers n and m such that $b^n x = b^m y$.

The relation ρ_b is a congruence relation on S and b is called the standard element determining the corresponding decomposition of S. Furthermore, for any b, S/ρ_b is a group; the congruence class modulo ρ_b containing b is the identity element of S/ρ_b and it is a subsemigroup of S. We call S/ρ_b the structure group of S with respect to b.

DEFINITION 2. Let S_α be an arbitrary congruence class of S (mod ρ_b). The following relation, $\geqq_b$ is a partial order on S_α. Let $x, y \in S_\alpha$. We define $\geqq_b$ on S_α by $x \geqq_b y$ if and only if there exists a positive integer n such that $y = b^n x$, or $y = x$.

DEFINITION 3. A discrete tree R is a lower semilattice (i.e., a