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A LINEAR TRANSFORMATION THEOREM FOR ANALYTIC FEYNMAN INTEGRALS

J. KUELBS

The behavior of the analytic Feynman integral under translation has been studied in the recent work of R. H. Cameron and D. A. Storvick. The purpose of this paper is to continue the development of this transformation theory. In particular, the behavior of the analytic Feynman integral under certain linear transformations is determined and, using this linear transformation theory, a "generalized Schroedinger equation" is solved in terms of an analytic Feynman integral.

2. **Preliminaries.** The analytic Wiener and analytic Feynman integrals were defined in [4] and [6] as follows:

DEFINITION 2.1. Let the complex number λ_0 satisfy $Re\lambda_0 \geqq 0$ and $\lambda_0 \neq 0$, so that $\lambda_0 = |\lambda_0| \exp(i\theta)$ for some θ on the interval $[-\pi/2, \pi/2]$. Let $F(x)$ be a functional defined on $C[a, b]$ (the space of continuous functions on $[a, b]$ which vanish at a) such that the Wiener integral

$$J(\lambda) = \int_{C[a,b]} F(\lambda^{-\frac{1}{2}}x)dx$$

exists for all real λ in the interval $|\lambda_0| < \lambda < |\lambda_0| + \delta$ for some $\delta > 0$. Then if $J(\lambda)$ can be extended so that it is defined and continuous on the closed region

$$S = \{\lambda = \rho e^{i\gamma} : |\lambda_0| \leqq \rho \leqq |\lambda_0| + (1 - \gamma\theta^{-1})\delta , \\ \gamma \in [0, \theta] \text{ or } \gamma \in [\theta, 0]\} , \tag{2.1}$$

and analytic in its interior, we define

$$\int_{C[a,b]}^{anw_{\lambda_0}} F(x)dx = J(\lambda_0) \tag{2.2}$$

and we call the left member of (2.2) the analytic Wiener integral of $F(x)$ with parameter λ_0. If $\theta = 0$ we interpret S to be the interval $[\lambda_0, \lambda_0 + \delta]$, omit the analyticity requirement since the interior of S is empty, and define the analytic Wiener integral to be $J(\lambda_0^+)$. If $\lambda_0 = -i$ the integral (2.2) will be called the analytic Feynman integral and we write

$$\int_{C[a,b]}^{anf} F(x)dx = J(-i) .$$