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LOCAL ANALYTIC EXTENSIONS OF THE RESOLVENT

JACK D. GRAY

Consider an endomorphism T, (that is, a bounded, linear transformation) on a (complex) Banach space X to itself. As usual, let $R(\lambda, T) = (\lambda 1 - T)^{-1}$ be the resolvent of T at $\lambda \in \rho(T)$. Then it is known that the maximal set of holomorphism of the function $\lambda \to R(\lambda, T)$ is the resolvent set $\rho(T)$. However, it can happen that for some $x \in X$, the X-valued function $\lambda \to R(\lambda, T)x$ has analytic extensions into the spectrum $\sigma(T)$ of T. Using this fact we shall, in § 1, localize the concept of the spectrum of an operator. In sections 2, 3 and 4 we investigate, quite thoroughly, the structural properties of this concept. Finally, in § 5, the results of the previous sections will be utilized to construct a local operational calculus which will then be applied to the study of abstract functional equations.

1. **The localization of the spectrum.** We begin by making the following remarks. For an X-valued function u to be analytic on some open subset K of a Riemann surface, it is necessary and sufficient that for each continuous linear functional x^* in some determining manifold for X, ([5], 34), the complex-valued function x^*u be analytic in K. K will be called the maximal set of analyticity of u, if each accessible point of the boundary of K is a singular point of u.

Now let T: $D_T \subseteqq X \to X$ be an arbitrary linear transformation on its domain D_T, and suppose that $x \in X$.

DEFINITION 1.1. The local resolvent set of T at x, $\rho(x, T)$, is that set of points $\zeta \in C$—the complex plane— for which there is a neighbourhood N of ζ, and an analytic function u: $N \to X$ which satisfies

$$\lambda u(\lambda) - Tu(\lambda) = x \tag{1.1}$$

for all $\lambda \in N$.

DEFINITION 1.2. The local spectrum $\sigma(x, T)$ of T at x is the complement in C of $\rho(x, T)$.

DEFINITION 1.3. The local spectral radius of T at x is given

$$r(x, T) = \sup_{\lambda \in \sigma(x,T)} |\lambda| .$$

DEFINITION 1.4. Any function u as given in Definition 1.1 will