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A NOTE ON CLT GROUPS

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Let A, B, C be respectively the class of all finite supersolvable groups, the class of all finite groups which satisfy the converse to Lagrange's theorem, and the class of all finite solvable groups. We show that $A \subset B \subset C$, and give examples to show that both of the inclusions are actually proper.

Throughout, 'n', 't', 'a_1', 'a_2', $\cdots$, 'a_t' will denote positive integers; 'p_1', 'p_2', $\cdots$, 'p_t' will denote pairwise distinct positive integer primes. If G and H are finite groups, then, 'G'' will denote the commutator subgroup of G, '$G \times H$' will denote the external direct product of G and H, and '$|G|$' will denote the order of G. 'A_4' will denote the alternating group on 4 symbols, 'e' will denote the identity of A_4, and 'C_2' will denote the cyclic group of order 2.

We are concerned here only with finite groups; throughout, when we say 'group', we intend this to be read as 'finite group', and 'G' will always denote a finite group. Our version of the converse to Lagrange's theorem is as follows:

DEFINITION. *G is a CLT group* if and only if for each d, the following holds: if d is a positive integer divisor of $|G|$, then G has at least one subgroup H with $|H| = d$.

All terminology not used in the above definition will be that of [2].

LEMMA 1. *$|G| = n = p_1^{a_1}p_2^{a_2}\cdots p_t^{a_t}$ and $n_i = n/p_i^{a_i}$ for $i = 1, 2, \cdots, t$. Then G is solvable if and only if G has subgroups with orders $n_1, n_2, \cdots, n_t$.*

Proof. This follows readily from Theorem 9.3.1, p. 141, and Theorem 9.3.3, p. 144 of [2].

LEMMA 2. *$|G| = n = p_1^{a_1}p_2^{a_2}\cdots p_t^{a_t}$ with $p_1 < p_2 < \cdots < p_t$. Then if G is supersolvable, G has normal subgroups with orders $1, p_t, p_t^2, \cdots, p_t^{a_t}$.*

Proof. This follows readily from Corollary 10.5.2, p. 159 of [2].

THEOREM 1. *Every CLT group is solvable.*

Proof. This is trivial if $|G| = 1$. Let G be a CLT group with $|G| = n = p_1^{a_1}p_2^{a_2}\cdots p_t^{a_t}$, and let $n_i = n/p_i^{a_i}$ for $i = 1, 2, \cdots, t$; since