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THE PRODUCT FORMULA FOR THE THIRD OBSTRUCTION

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Let ξ be an $SO(n)$-bundle with $n > 3$; let $p: E \to B$ be the projection in the associated $(n-1)$-sphere bundle. In this note we express the third obstruction to a cross-section of p as a tertiary characteristic class and prove a product formula for the behavior of this class under Whitney sum.

The first obstruction is the Euler class $\chi(\xi) \in H^n(B; Z)$. χ is a primary characteristic class and satisfies $\chi = j^*(U)$, where $j: B \to T$ is the inclusion into the Thom space and $U \in H^n(T; Z)$ is the Thom class. Whenever $\chi(\xi) = 0$, a secondary characteristic class

$$\alpha(\xi) \in H^{n+1}(B; Z_2)/(Sq^2 + w_2 \smile)H^{n-1}(B; Z)$$

is defined. α is the second obstruction and satisfies

$$\alpha = (Sq^2 + w_2 \smile)_j(U) .$$

Thus α is obtained by applying a twisted functional primary operation to U. The third obstruction $\gamma(\xi)$, defined whenever $\alpha(\xi) \equiv 0$, will be expressed as the value $\Phi_j(U)$ of a certain twisted functional secondary operation.

It is immediately plausible to consider as $(n+1)$-ary characteristic classes the values of certain functional twisted n-ary operations on U, defined when appropriate n-ary characteristic classes vanish. We hope to deal with such classes systematically in a future paper, but the treatment is expected to be more complicated technically; hence $\gamma(\xi)$ is presented here as an illustrative example in a straightforward setting.

The paper is organized as follows. Section 2 is a statement of results, while in § 3 we define $\gamma(\xi)$. The Peterson-Stein formula and the proof of (2.2) appears in § 4; the product formula is obtained in § 5. We conclude in § 6 with an example.

Throughout the paper all cohomology is taken with Z_2 as coefficients unless otherwise indicated.

2. Statement of results. Suppose ξ is an $SO(n)$-bundle with $n > 3$ and suppose $\chi(\xi) = 0$. Let

$$\alpha(\xi) \in H^{n+1}(B)/(Sq^2 + w_2 \smile)H^{n-1}(B; Z)$$

be the secondary characteristic class given by $\alpha(\xi) = (Sq^2 + w_2 \smile)_j(U)$