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CONVOLUTION TRANSFORMS WHOSE INVERSION FUNCTION HAS COMPLEX ROOTS IN A WIDE ANGLE

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In this paper a class of convolution transforms:

$$f(x) = \int_{-\infty}^{\infty} G(x-t)\varphi(t)dt \tag{1.1}$$

whose kernels $G(t)$ satisfy

$$G(t) = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} [E(s)]^{-1} \cdot e^{st} ds \tag{1.2}$$

where

$$E(s) = \prod_{k=1}^{\infty}(1 - s/a_k) \quad \text{or} \quad E(s) = \prod_{k=1}^{\infty}(1 - s/a_k)\exp(sRe\, a_k^{-1}) \tag{1.3}$$

will be treated. Investigation of properties will be carried for the subclass defined by the restriction on a_k as follows:

(a) **For some ψ, $0 < \psi < \pi/2$**

$$\min_{n=0,1,2} |n\pi - \arg a_k| \leqq \psi \quad \text{where} \quad a_k \neq 0 .$$

(b) **For some $0 < q < 1$**

$$\text{and integer } l \quad |a_{k+l}| \geqq q^{-1}|a_k| \quad \text{for all} \quad k \geqq k_0 .$$

It should be mentioned that the restriction (a) on the argument of a_k is much weaker than those used in other subclasses of convolution transforms defined by (1.1), (1.2) and (1.3) that were investigated before.

I. I. Hirschman and D. V. Widder [4] treated a class of transforms for which $\arg a_k$ tend to either 0 or π. J. Dauns and D. V. Widder [1] and the author [2] studied the case $E(s) = \prod_{k=1}^{\infty}(1 - s^2/a_k^2)$ for which $|\arg a_k| \leqq \psi < \pi/4$, that is: The sequence of roots contains pairs of a_k and $-a_k$. A milder way of coupling was introduced by the author [3]. The question that arises is: Can we relax the restriction on the argument of the a_k's and still have the transforms and their inversion formulae? Of course it was shown [1, p. 442] that in some simple cases the analogous inversion formula to that of Hirschman and Widder does not hold. Examples can be given to show that in some cases (1.2) does not converge. Here a restriction on the growth of the roots is given (b) which assures us of the convergence of (1.2) and helps us to prove that