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ALGEBRAS SATISFYING THE DESCENDING CHAIN CONDITION FOR SUBALGEBRAS

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In this paper we give a partial solution to the following problem of B. Jónsson:

(*) For which cardinals m do there exist algebras of power m having finitely many operations and satisfying the descending chain condition for subalgebras?

Of course a necessary condition for the existence of such an algebra is that there exist an algebra of power m having finitely many operations and having no proper subalgebra of power m. The first such construction was by F. Galvin who constructed an algebra of power ω_1 which satisfied the descending chain condition for subalgebras. It has been shown by Erdos and Hajnal [1] that for $n \in \omega$ there is an algebra of power ω_n which has finitely many operations and has no proper subalgebra of power ω_n. Actually C. C. Chang [3] has shown that if an algebra exists of power m having finitely many operations and having no proper subalgebra of power m, then such an algebra exists of power m^+. In § 2 we modify this construction to show that if there is an algebra of power m with finitely many operations and satisfying the descending chain condition, then there is such an algebra of power m^+.

Erdos and Hajnal [1] also showed, under the assumption of the generalized continuum hypothesis, that for any cardinal m there is a locally finite algebra of power m^+ having finitely many operations and having no proper subalgebra of power m^+. In § 3 we show that for $n \in \omega$ there is a locally finite algebra of power ω_n having finitely many operations and satisfying the descending chain condition for subalgebras.

2. General algebras.

Before beginning the construction of the algebras we note the following relevant theorem of W. Hanf.

THEOREM 2.1. (Hanf [2], [4]). *The lattice of subalgebras of an algebra with countably many operations is a compactly generated lattice in which each compact element contains at most countably many compact elements. Conversely, any such lattice can be realized as the lattice of subalgebras of a commutative loop in which each subalgebra is a subloop.*

COROLLARY 2.2. *The following are equivalent:*

(i) *There exists a compactly generated lattice having m compact elements in which each compact element contains at most countably*