PACIFIC JOURNAL OF MATHEMATICS
Vol. 28, No. 1, 1969

ON H-EQUIVALENCE OF UNIFORMITIES (II)

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This paper, continuing previous work by the same author, is concerned with the following problem: Given a metrisable uniformity $\mathfrak{U}$ for a set X, does there exist another (distinct) uniformity $\mathfrak{V}$ for X such that the two corresponding Hausdorff uniformities induce the same topology on the set, $S(X)$ say, of all nonempty subsets of X? Sufficient conditions for the existence, and sufficient conditions for the nonexistence, of such a uniformity $\mathfrak{V}$ are given, together with related results concerning the Hausdorff uniformities (derived from $\mathfrak{U}$ and $\mathfrak{V}$) for $S(X_1)$, where X_1 is a subset of X, everywhere dense in the topology derived from $\mathfrak{U}$.

The notation is that used in the previous paper [4]; Theorem 1 of that paper will be referred to as Theorem 1A, and so on. We shall also say for brevity that a uniformity $\mathfrak{V}$ is *H-singular* (over X) if and only if there exists no distinct uniformity for X which is H-equivalent to $\mathfrak{V}$ on X.

1. *H*-equivalence on dense subsets. Our first theorem will allow an improvement of Theorem 4A.

THEOREM 1. *Let $\mathfrak{V}$ be a metrisable uniformity for X (that is, one with an enumerable base in $X \times X$) and X_1 a subset dense in X, in the topology $\mathscr{T}(\mathfrak{V})$ induced by $\mathfrak{V}$. Let $\mathfrak{U}$ be another uniformity for X, such that*

(a) $\mathscr{T}(\mathfrak{U}) \subset \mathscr{T}(\mathfrak{V})$ on X;

(b) *the restrictions $\mathfrak{U}_1$, $\mathfrak{V}_1$ of $\mathfrak{U}$, $\mathfrak{V}$ to $X_1 \times X_1$ are H-equivalent on X_1.*

Then if $\mathfrak{U}$ and $\mathfrak{V}$ are not H-equivalent on X the cardinal of X must be measurable.

We achieve the proof by five propositions, the first two of which do not depend on the metrisability of $\mathfrak{V}$.

(i) $\mathfrak{U} \subset \mathfrak{V}$.

By Theorem 1A[1], $\mathfrak{U}_1$ and $\mathfrak{V}_1$ are proximity-equivalent (on X_1); as $\mathfrak{V}_1$ is metrisable this implies $\mathfrak{U}_1 \subset \mathfrak{V}_1$. Given $U_0 \in \mathfrak{U}$, take a symmetric $U \in \mathfrak{U}$ such that $\overset{3}{U} \subset U_0$, and a symmetric $V \in \mathfrak{V}$ such that $\overset{3}{V} \cap (X_1 \times X_1)$

[1] The part of Theorem 1A actually used here was proved earlier by D. H. Smith, [**1**, Th. 1].