PACIFIC JOURNAL OF MATHEMATICS
Vol. 28, No. 1, 1969

VECTOR VALUED ORLICZ SPACES GENERALIZED N-FUNCTIONS, I.

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The theory of Orlicz spaces generated by N-functions of a real variable is well known. On the other hand, as was pointed out by Wang, this same theory generated by N-functions of more than one real variable has not been discussed in the literature. The purpose of this paper is to develop and study such a class of generalized N-functions (called GN-functions) which are a natural generalization of the functions studied by Wang and the variable N-functions by Portnov. In second part of this study we will utilize GN-functions to define vector-valued Orlicz spaces and examine the resulting theory.

This paper is divided into five sections. In § 2, we define and examine some basic properties of GN-functions. A generalized delta condition is introduced and characterized in § 3. In § 4 and § 5 we present, respectively, the theory of an integral mean for GN-functions and the concept of a conjugate GN-function. A complete bibliography on Orlicz spaces, N-functions, and related material can be found in [4, 8]. The study of variable N-functions by Portnov can be found in [6, 7] and the study of nondecreasing N-functions by Wang in [9].

2. GN-functions.

In what follows T will denote a space of points with σ-finite measure and E^n n dimensional Euclidean space.

DEFINITION 2.1. Let $M(t, x)$ be a real valued nonnegative function defined on $T \times E^n$ such that

(i) $M(t, x) = 0$ if and only if $x = 0$ for all $t \in T, x \in E^n$,

(ii) $M(t, x)$ is a continuous convex function of x for each t and a measurable function of t for each x,

(iii) For each $t \in T$, $\lim_{|x|=\infty} \dfrac{M(t, x)}{|x|} = \infty$, and

(iv) There is a constant $d \geqq 0$ such that

$$(*) \qquad \inf_t \inf_{c \geqq d} k(t, c) > 0$$

where

$$k(t, c) = \frac{\underline{M}(t, c)}{\bar{M}(t, c)}, \ \bar{M}(t, c) = \sup_{|x|=c} M(t, x) \ ,$$

$$\underline{M}(t, c) = \inf_{|x|=c} M(t, x)$$