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ON INTERPOLATION OF q-VARIATE STATIONARY STOCHASTIC PROCESSES

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Let X_t be a q-variate stationary stochastic process. Let K be any set of t-values and let K' be the complement of K. If $s \in K'$ the problem of approximating X_s by linear combinations of the X_t's with $t \in K$ and limit of such linear combinations is considered. The best linear predictor and the mean square error matrix are evaluated in the following cases: (1) t takes on all real values, K consists of the integers (2) t is interger-valued, K consists of the odd integers.

Let $(X_k)_{-\infty}^{\infty}$, k an integer, be a q-variate weakly stationary stochastic process (SP). Let K be any subset of the set of integers and K' denote its complement in the set of all integers. Let $\mathscr{M}_K$ denote the (closed) subspace spanned by X_k, $k \in K$.

PREDICTION PROBLEM. Let X_s, $s \in K'$. Find $\hat{X}_s$ the projection of X_s onto $\mathscr{M}_K$ and the error matrix $(X_s - \hat{X}_s, X_s - \hat{X}_s)$[1].

In this paper we propose to solve the prediction problem for two cases:

(1) X_t, t real, is a q-variate stationary SP and K consists of the set of all integers.

(2) X_k, k an integer, is a q-variate stationary SP and K consists of the set of all odd integers.

For $q = 1$ these results have been previously obtained by A. M. Yaglom {cf. [12, p. 176]}.

In § 2 we will review the notion of absolute continuity of a matrix-valued signed measure with respect to another such measure {cf. [6]} and state a few results concerning the Hellinger-square integrability of matrix-valued measures. Our main result will be given in § 3.

2. Matrix-valued measures. The problem of absolute continuity of a matrix-valued measure with respect to another matrix-valued measure was first posed by P. Masani in [4, p. 366]. Later J. B. Robertson and M. Rosenberg {cf. [6]} dealt with this question and were able to obtain a satisfactory solution to it. We will briefly review some of these results. Let Ω be any set and $\mathscr{B}$ be a σ-algebra of its subsets. M is said to be a $q \times r$ matrix-valued signed measure on $(\Omega, \mathscr{B})$ if for each $B \in \mathscr{B}$, $M(B)$ is a $q \times r$ matrix, with finite complex

[1] $(\ldots)$ denotes the inner product in the Hilbert space $\mathscr{H}^q$ containing the q-variate stochastic process X_k, k an integer.