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SOME 5/2 TRANSITIVE PERMUTATION GROUPS

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In this paper we classify those 5/2-transitive permutation groups $\mathfrak{G}$ such that $\mathfrak{G}$ is not a Zassenhaus group and such that the stabilizer of a point in $\mathfrak{G}$ is solvable. We show in fact that to within a possible finite number of exceptions $\mathfrak{G}$ is a 2-dimensional projective group.

If p is a prime we let $\Gamma(p^n)$ denote the set of all functions of the form

$$x \longrightarrow \frac{ax^\sigma + b}{cx^\sigma + d}$$

where $a, b, c, d \in GF(p^n)$, $ad - bc \neq 0$ and σ is a field automorphism. These functions permute the set $GF(p^n) \cup \{\infty\}$ and $\Gamma(p^n)$ is triply transitive. Moreover $\Gamma(p^n)_\infty = S(p^n)$, the group of semilinear transformations on $GF(p^n)$. Let $\bar{\Gamma}(p^n)$ denote the subgroup of $\Gamma(p^n)$ consisting of those functions of the form

$$x \longrightarrow \frac{ax + b}{cx + d}$$

with $ad - bc$ a nonzero square in $GF(p^n)$. Thus $\bar{\Gamma}(p^n) \cong PSL(2, p^n)$.

Let $\mathfrak{G}$ be a permutation group on $GF(p^n) \cup \{\infty\}$ with $\Gamma(p^n) \supseteqq \mathfrak{G} > \bar{\Gamma}(p^n)$. Since $\bar{\Gamma}(p^n)$ is doubly transitive so is $\mathfrak{G}$. Now $\Gamma(p^n)/\bar{\Gamma}(p^n)$ is abelian so $\mathfrak{G}$ is normal in $\Gamma(p^n)$. Hence $\mathfrak{G}_{\infty 0} \triangle \Gamma(p^n)_{\infty 0}$. Since a nonidentity normal subgroup of a transitive group is half-transitive we see that $\mathfrak{G}_{\infty 0}$ is half-transitive on $GF(p^n)^\#$ and hence $\mathfrak{G}$ is 5/2-transitive. It is an easy matter to decide which group $\mathfrak{G}$ with $\Gamma(p^n) \supseteqq \mathfrak{G} > \bar{\Gamma}(p^n)$ are Zassenhaus groups. If $p = 2$, there are none while if $p > 2$, we must have $[\mathfrak{G} : \bar{\Gamma}(p^n)] = 2$. In this latter case, there is one possibility for n odd and two for n even. The main result here is:

THEOREM. *Let $\mathfrak{G}$ be a 5/2-transitive group which is not a Zassenhaus group. Suppose that the stabilizer of a point is solvable. Then modulo a possible finite number of exceptions we have, with suitable identification, $\Gamma(p^n) \supseteqq \mathfrak{G} > \bar{\Gamma}(p^n)$ for some p^n.*

The question of the possible exceptions will be discussed briefly in § 3. We use here the notation of [4]. Thus we have certain linear groups $T(p^n)$ and $T_0(p^n)$ and certain permutation groups $S(p^n)$