PACIFIC JOURNAL OF MATHEMATICS
Vol. 28, No. 1, 1969

REARRANGEMENT OF SPHERICAL MODIFICATIONS

M. V. MIELKE

A "rearrangement" theorem of Wallace states essentially that if a manifold M is the trace of a sequence of spherical modifications of various types then these modifications can be arranged so that the order in which they are performed is that of increasing type, their trace still being M. In this paper a related rearrangement problem is considered; namely, to determine bounds on how "mixed" the order of performing a sequence of modifications can be and still possess the same trace M.

Sections 2 through 4 are concerned with basic definitions and preliminary results. The most important of which is the establishment of an algorithm to determine a measure of how "mixed" the order of a sequence of integers is [§ 4]. The main results appear in § 5.

2. Spherical modifications.

Unless stated otherwise an n-manifold is a compact, differentiable n-dimensional manifold without boundary.

Let V_1 be an n-manifold and suppose S^i is an i-sphere homeomorphically and smoothly imbedded in V_1 with a trivial normal bundle. Then S^i has a neighborhood of the form $S^i \times D^{n-i}$. (D^{n-i} is an $(n-i)$ − disc). Clearly the boundary of $S^i \times D^{n-i} = S^i \times S^{n-i-1} =$ the boundary of $D^{i+1} \times S^{n-i-1}$. Smoothly identifying the boundary of $D^{i+1} \times S^{n-i-1}$ with the boundary of (V_1-interior $(S^i \times D^{n-i})$) results in a new manifold V_2. V_2 is said to be obtained from V_1 by a spherical modification of type i, or by an i-type modification. ([7], p. 504).

Associated to the spherical modification is an $n+1$-manifold W called the trace of the modification. The boundary of $W = V_1 \cup V_2$ and the triple $(W; V_1, V_2)$ is a manifold triad in the sense of [4] page 2. As a matter of convention, performing a type-1 modification on V_1 is taken to mean $V_1 = \varnothing$, $V_2 =$ an n-sphere, and the trace is an $n+1$ − disc. The n will be clear from context. For a further discussion of the trace see [8] page 775.

3. Realizable sequences.

DEFINITION 3.1. An admissible sequence $S(n)$ is a finite sequence of integers $a_1, a_2, \cdots, a_l$ with $a_1 = -1, a_l = n-1, -1 \leqq a_i \leqq n-1$ for $i = 1, 2, \cdots, l$ and $a_i \neq a_j$ if $i \neq j$.

DEFINITION 3.2. Let $S(n)$ be an admissible sequence. $S(n)$ is re-