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ON CROISOT'S THEORY OF DECOMPOSITIONS

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Croisot gave a definition of (m, n)-regularity which he then showed defined four logically distinct classes of semi-groups. However, semigroups with nilpotent elements did not fall within his classification. Our generalization of $(m, n)^0$-regularity remedies this exclusion; countably many distinct classes of semi-groups are defined.

In particular we investigate the structure of semigroups which are $(2, 2)^0$-regular. We show that a semigroup S is in this class precisely when for each $x \in S$ either $x^2 = 0$ or $x^2 \in H_x$. Further, each regular $\mathscr{D}$-class together with 0 of such a semigroup is itself a completely 0-simple semigroup. The $(2, 2)^0$-regularity condition is specialized to that of absorbency: for each $a, b \in S$ either $ab = 0$ or $ab \in (R_a \cap L_b)$. We show that a regular absorbent semigroup is just a mutually annihilating collection of completely 0-simple semigroups.

A schematic summary of the two classifications is found in Fig. 1 and Fig. 2. We remark now that our classification provides a countable number of open problems; e.g., determining the structure of $(n, n)^0$-regular semigroups for $n > 2$. Moreover, it also remains to treat such $(m, n)^0$-regular semigroups with reciprocity, antireciprocity, and uniqueness conditions (cf. [1], p. 124 or [2], p. 373 ff.).

We finally show that for an absorbent semigroup without 0 Green's relations $\mathscr{L}$ and $\mathscr{R}$ are congruences. It is then shown that a regular simple semigroup S is completely simple if and only if $\mathscr{L}$ and $\mathscr{R}$ are congruence relations on S.

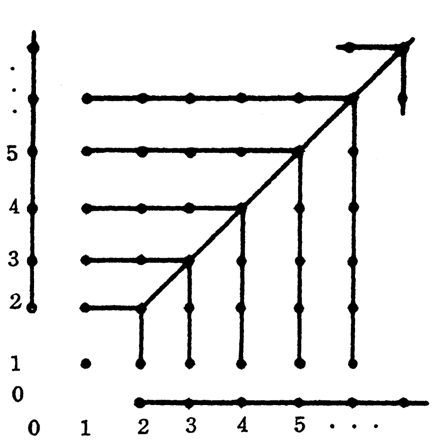

FIG. 1—(m, n)-regular classes

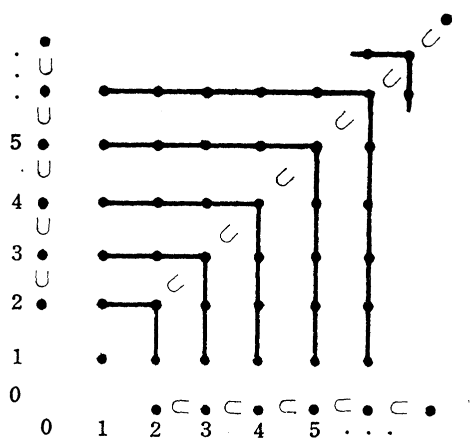

FIG. 2—$(m, n)^0$-regular classes