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THE SCHUR INDEX FOR PROJECTIVE REPRESENTATIONS OF FINITE GROUPS

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In this paper the question of determining the absolutely irreducible constituents of an irreducible projective representation of a finite group is considered from the viewpoint of the theory of algebras. New proofs are given for several of the main results of the theory of representations of finite dimensional associative algebras. This theory is applied to determine the center of a simple component of a twisted group algebra modulo its radical and sufficient conditions are given to insure that this center is a normal extention of the base field. The Schur index of an absolutely irreducible projective representation of a finite group is defined as in the theory of linear representations of finite groups. It is shown that every irreducible complex projective representation of a finite group is projectively equivalent to a representation whose associated factor system has values which are roots of unity but whose Schur index over the rationals is 1.

Throughout this paper all modules will be assumed to be finite dimensional unitary left modules. By an algebra over a field K we shall mean a finite dimensional associative algebra in which the identity of K acts as identity. If $\mathfrak{A}$ is an algebra, we denote the radical of $\mathfrak{A}$ by rad $\mathfrak{A}$ and set $\bar{\mathfrak{A}} = \mathfrak{A}/\text{rad}\,\mathfrak{A}$. If E is an extension field of K, we identify $\mathfrak{A}$ as a subalgebra of $\mathfrak{A}^E = E \otimes_K \mathfrak{A}$. If M is an $\mathfrak{A}$-module, we denote by M^E the $\mathfrak{A}^E$-module, $E \otimes_K M$. We refer the reader to [4] for the relevant theory of algebras assumed.

1. The Schur index.

Let $\mathfrak{A}$ be an algebra over the perfect field K. There exists a finite normal (separable) extension field E of K which is a splitting field for $\mathfrak{A}$, i.e., $\mathfrak{A}^E/\text{rad}\,(\mathfrak{A}^E)$ is a direct sum of complete matrix rings over E. Let $\{x_1, \cdots, x_s\}$ be a basis for $\mathfrak{A}$ over K. Under the usual identification, $\{x_1, \cdots, x_s\}$ is also a basis for $\mathfrak{A}^E$. We denote the Galois group of E over K by $\mathfrak{G}(E \mid K)$. Let N be a left $\mathfrak{A}^E$-module with basis $\{m_1, \cdots, m_n\}$ over E and let the module action be given by $x_k m_i = \sum_j a_{ij}(x_k) m_j$, $k = 1, \cdots, s$, $a_{ij}(x_k) \in E$. Let V be an n-dimensional vector space over E with basis $\{v_1, \cdots, v_n\}$ and let $\sigma \in \mathfrak{G}(E \mid K)$. Under the action $x_k v_i = \sum_j \sigma(a_{ij}(x_k)) v_j$, $k = 1, \cdots, s$, V becomes an $\mathfrak{A}^E$-module which we denote by σN. σN is called a conjugate module to N and the representations of $\mathfrak{A}^E$ that these modules afford are said to be conjugate (with respect to the pair of bases $\{x_1, \cdots, x_s\}$, $\{v_1, \cdots, v_s\}$). By the character afforded by a representation