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NONOSCILLATORY SOLUTIONS OF SECOND ORDER NONLINEAR DIFFERENTIAL EQUATIONS

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We consider here a generalization of the equation

$$x'' + a(t)x^{2n+1} = 0$$

where $a(t)$ is a continuous non-negative function on $[0, +\infty)$ and $n \geqq 0$ is an integer. Necessary and sufficient conditions are given for the existence of

(1) a bounded nonoscillatory solution with prescribed limit at ∞;

(2) a nonoscillatory solution whose derivative has a positive limit at ∞.

Specifically, we are concerned with the asymptotic behavior of the solutions of the following second order nonlinear differential equation:

$$x'' + f(t, x)g(x') = 0 . \tag{1}$$

We shall assume the following conditions hold:

(A_0) $f(t, x)$, $g(x')$, and the partial derivative function $f_x(t, x)$ are all continuous for $t \geqq 0$, $x' \geqq 0$, and $|x| < +\infty$.

(A_1) $f(t, 0) = 0,\ t \geqq 0 .$

(A_2) $f_x(t, x) \geqq 0$ and is nondecreasing in x for $t \geqq 0$ and $x \geqq 0$.

(A_3) $g(x') > 0$ for all $x' \geqq 0$.

As a special case we have the equation

$$x'' + a(t)x^{2n+1} = 0,\ n \geqq 0 , \tag{2}$$

in which $a(t) \geqq 0$ for $t \geqq 0$ and $g(x') = 1$ for all x'. Oscillatory and nonoscillatory properties of (2) for the case $n \geq 1$ were investigated by Atkinson in [1], Moore and Nehari in [5], and Utz in [9]. Generalizations of equation (2) have been considered by Waltman in [7] and [8], Nehari in [6], Wong in [10], and Macki and Wong in [4].

We shall study equation (1) by considering the equation

$$x'' + f_x(t, \alpha)x = 0 , \tag{3}$$

where α is some real constant depending on solutions of (1). To do this we shall need to establish several lemmas concerning the equation