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ON THE DECOMPOSITION OF INFINITELY DIVISIBLE PROBABILITY LAWS WITHOUT NORMAL FACTOR

ROGER CUPPENS

In the theory of the decomposition of probability laws, the fundamental problem stated by D. A. Raikov of the characterization of the class I_0 of the infinitely divisible laws without indecomposable factors has been studied in the case of univariate laws by Yu. V. Linnik and I. V. Ostrovskiy. Lately, we have shown that nearly all these results can be extended to the case of multivariate laws. In this paper, we give a result which can be considered as an extension of a theorem of Raikov and P. Lévy and of a particular case of theorems of Linnik, and the extension of this result to the case of several variables.

If we consider the finite products of Poisson laws, i.e., the characteristic functions of the variable t of the form

$$f(t) = \exp\left\{ict + \sum_{j=1}^{p} \lambda_j[\exp(i\alpha_j t) - 1]\right\}$$

(c real, $\lambda_j > 0$, $\alpha_j > 0$), three general results are known, the first being owed to D. A. Raikov [9] and P. Lévy [4] and the third to Yu. V. Linnik [5, Chapter 9]:

(a) if $\alpha_1, \cdots, \alpha_p$ are rationally independent, f has no indecomposable factor;

(b) if $\alpha_1, \cdots, \alpha_p$ are such that $0 < a \leqq \alpha_j \leqq 2a$ $(j = 1, \cdots, p)$, f has no indecomposable factor;

(c) if α_{j+1}/α_j is an integer greater than 1 $(j = 1, \cdots, p-1)$, f has no indecomposable factor.

Lately, I. V. Ostrovskiy [8] has extended the two results (a) and (b) of Raikov and Lévy to the case of a continuous spectrum, the base of his study being the

THEOREM 1. (see also [1] chapter 8). *Let f_0 be the infinitely divisible characteristic function of the variable t defined by*

$$f_0(t) = \exp\{i\gamma t + \int_a^b [\exp(ixt) - 1]d\mu(x)\},$$

where γ is a real constant and μ is a nonnegative measure defined on the segment $[a, b]$ $(0 < a < b < \infty)$. If f_1 is a factor of f_0, then

$$f_1(t) = \exp\left\{ict + \int_a^b [\exp(ixt) - 1]dm(x)\right\},$$