A THEOREM ON SEQUENTIAL CONVERGENCE OF MEASURES AND SOME APPLICATIONS

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If S is a locally compact Hausdorff space, let βS be its Stone-Cech compactification and let $M(S)$ be the space of all finite complex valued regular Borel measures on S. In this paper we will prove that whenever S is paracompact and $\{\mu_n\}$ is a sequence in $M(\beta S)$ which converges to zero in the weak star topology, then $\lim \int_S f d\mu_n = 0$ for every continuous function f, and $\{\mu_n\}$ satisfies a certain uniformity condition on S. This generalizes a result of R. S. Phillips on weak star sequential convergence in the dual of l^∞. Moreover, by using our theorem we can obtain many previously known results whose proofs, though all similar, were apparently independent.

Indeed, the motivation for undertaking the research which led to this paper came from the similarity of the proofs of a number of theorems. Among these is the following.

THEOREM A. *If S is paracompact and H is a subset of $M(S)$ which is countably compact in the $C(S)$ topology on $M(S)$ then for every $\varepsilon > 0$ there is a compact set $K_\varepsilon \subset S$ such that*

$$|\mu|(S - K_\varepsilon) < \varepsilon$$

for every μ in H.

This theorem was first proved by LeCam [15] and then, independently, by the present author [4]. Another result which utilizes the same method of proof referred to above is

THEOREM B. *If S is paracompact and A is a closed linear subspace of $C(\beta S)$ such that A interpolates $\beta S - S$ then there is a closed neighborhood V of $\beta S - S$ such that A interpolates V.*

NOTE. To say that a closed linear subspace A of $C(\beta S)$ interpolates a closed set $K \subset \beta S$ means that any f in $C(K)$ has an extension F to βS such that F is in A.

Theorem B is due to Bade [1] for σ-compact spaces. In addition to these two results there are still others which fit into this same category. These will be presented in § 3 together with the proofs using the main theorem of this paper. However, we have not yet been able to discover a proof of Theorem A based on this result.