PACIFIC JOURNAL OF MATHEMATICS
Vol. 28, No. 1, 1969

EQUATIONAL CLASSES OF MODULAR LATTICES

KIRBY A. BAKER

One natural question of lattice theory has been (i) whether there exists an equational class of lattices which cannot be characterized by any finite list of lattice identities. Another question, due to B. Jónsson, is (ii) whether there exists an equational class of lattices which is not determined by its finite members. We shall show that the answers to both questions are affirmative, even with the additional requirement of modularity. The examples are constructed from lattices corresponding to projective planes.

Using different methods, R. McKenzie [9] has independently answered the first question (without modularity).

1. Stable classes of lattices. By an *equational class* of lattices is meant the class of all lattices satisfying some fixed finite or infinite set of lattice identities. Birkhoff [2] has shown for abstract algebras in general that a class of algebras with the "same" operations is an equational class if and only if it is closed under the formation of direct products, subalgebras, and homomorphic images. Jónsson [7] has sharpened this result in the case of algebras whose lattices of congruence relations are distributive; we shall merely state his key lemma, for the case of lattices. If $\mathscr{K}$ is any class of lattices, let $\mathscr{K}^e$ be the equational class of lattices generated by $\mathscr{K}$, i.e., the class of all lattices satisfying all lattice identities true in all lattices of $\mathscr{K}$. For further terminology, see Birkhoff [3].

LEMMA 1.1. (*Jónsson* [7], Corollary 3.2). *If $\mathscr{K}$ is a class of lattices and if a subdirectly irreducible lattice L is in $\mathscr{K}^e$, then L is a homomorphic image of a sublattice of an ultraproduct of lattices from $\mathscr{K}$.*

The following concept will be useful.

DEFINITION 1.2. A class $\mathscr{K}$ of lattices is *stable* if $\mathscr{K}$ is closed under the formation of sublattices, homomorphic images, and ultraproducts.

Clearly, any equational class of lattices is stable. Less trivial examples are provided by the following fact.

LEMMA 1.3. *Let P be a finite partially ordered set, and let*