PACIFIC JOURNAL OF MATHEMATICS
Vol. 28, No. 1, 1969

# ON THE GEOMETRY OF THE UNIT BALL IN THE SPACE OF REAL ANNIHILATING MEASURES

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**Our purpose is to study the geometry of the unit ball in the space of real measures on the boundary of a finite Riemann surface that annihilate the analytic functions on that surface with continuous boundary values. We show that the number of boundary components of the surface (and hence the topological type) can be determined from the geometry of this unit ball. More precisely, if the surface has $k$ boundary components then this unit ball has $2^k - 2$ "flat faces" of highest possible dimension. We also get some information on extreme points and conclude with an example to show that the linear structure of this unit ball depends not only on the topology of the surface but also on some of its conformal structure.**

Let $R$ be a finite Riemann surface with boundary $\Gamma$, and let $A$ denote the algebra of complex valued functions that are continuous on $\bar{R} = R \cup \Gamma$ and holomorphic on $R$, and let Re$A$ denote the space of real parts of functions in $A$. If $n$ is the first Betti number of $R$ then it is well-known that Re$A$ has codimension $n$ in $C_R(\Gamma)$, the space of continuous real valued functions on $\Gamma$. In other words, the space $N$ of real measures on $\Gamma$ that annihilate $A$ has dimension $n$ over the real numbers.

Fix $\xi \in R$ and let $dm$ denote the unique positive measure on $\Gamma$ such that

$$u(\xi) = \int u dm$$

for all $u$ that are continuous on $\bar{R}$ and harmonic on $R$. We know the following, from [4]: if $d\nu \in N$ then $d\nu = hdm$ where $h$ is the restriction to $\Gamma$ of a function that is meromorphic on $\bar{R}$ and regular on $\Gamma$. Moreover there is a meromorphic differential $\omega$ on $\bar{R}$ that is regular and nonvanishing on $\Gamma$ and real along $\Gamma$, $\omega$ has the property that if $hdm \in N$, $h\omega$ is holomorphic on $\bar{R}$. Also "$\omega = dm$" in the sense that

$$\int u dm = \int_\Gamma u\omega$$

for every continuous function $u$ on $\Gamma$. Hence if $h$ is meromorphic on $\bar{R}$ and real along $\Gamma$ then $hdm \in N$ because for all $f \in A$

$$\int f h dm = \int_\Gamma f h\omega = 0$$