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ON APPROXIMATION BY DILATIONS OF DISTRIBUTIONS

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Let E be a locally convex space of temperate distributions and suppose that $u \in E$. We attempt to characterize the closed vector subspace of E generated by the set of all distributions having the form $u(a_1x_1 + b_1, \cdots, a_nx_n + b_n)$ where $a_1, \cdots, a_n$, $b_1, \cdots, b_n$ are real numbers with $a_1, \cdots, a_n$ being nonzero. The characterization is effected in the case when the topology on E satisfies certain conditions.

1. Notation. The underlying topological group in all our analysis is the additive group R^n with the usual topology. Addition and multiplication in R^n, are defined component-wise, in the usual manner. We identify the Pontryagin character group of R^n with R^n. Typical elements of R^n will be denoted by $x, y, \cdots$, or, when we are thinking of R^n as its own character group, by $\chi, \xi, \cdots$. If $\chi \in R^n$, then the bounded continuous character of R^n corresponding to χ is defined by

$$(1.1) \qquad (x_1, \cdots, x_n) \rightarrow \exp\{-2\pi i(\chi_1 x_1 + \cdots + \chi_n x_n)\} .$$

The ordinary Lebesgue measure on R^n is denoted by dx, or by $d\chi$ if we think of R^n as its own character group. With the identification expressed in (1.1), the Fourier Inversion Formula holds without any multiplicative constants.

Throughout, we adopt the usual conventions and notations of the calculus of n variables; see, for example, Hörmander ([5], p. 4). If $x \in R^n$, and $k \leqq n$ is a positive integer, we write x_k for the k-th component of x. If α is a multi-index, then the function j^α on R^n is defined by $j^\alpha(x) = x_1^{\alpha_1} \cdots x_n^{\alpha_n}$ for all $x \in R^n$.

$R^{\neq}$ will denote the set $R^n \backslash \{x \in R^n : x_k = 0 \text{ for some } k\}$.

Let W be an open set in R^n. We write $C^\infty(W)$ for the set of all complex valued functions which are defined in W and are indefinitely differentiable there. $D(W)$ will denote the set of functions which are indefinitely differentiable and have compact support in W. The space of distributions with support in W is denoted by $D'(W)$. For an account of these spaces, see Schwartz [6] and [7].

The space of rapidly decreasing indefinitely differentiable functions on R^n is designated by $S(R^n)$. The topological dual $S'(R^n)$ of $S(R^n)$ is the space of temperate distributions on R^n. We shall always assume that $S'(R^n)$ is equipped with the strong topology $\beta(S', S)$.

Finally, let $\varphi \in D(R^n)$ and suppose that $b \in R^n$. Then the function $\varphi_b \in D(R^n)$ defined by