PACIFIC JOURNAL OF MATHEMATICS
Vol. 28, No. 3, 1969

ON NORMALOID OPERATORS

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The purpose of the present paper is to extend an earlier theorem of the author's on hyponormal operators to the following, on normaloid operators.

THEOREM. *Let N be an operator such that $N - zI$ is normaloid for all complex values of z. If $AN = N^*A$, for an arbitrary operator A, for which $0 \notin \mathrm{Cl}(W(A))$, then $N = N^*$.*

2. **Notations.** We consider bounded linear operators defined on a Hilbert space H. As usual, the symbols $s(T), \Sigma(T), W(T)$ and $\mathrm{Cl}(W(T))$ stand for the spectrum of an operator T, the closed convex hull of $s(T)$, the numerical range of T and the closure of $W(T)$ respectively.

An operator T is said to be normaloid if $\|T\| = \sup\{|z|; z \in s(T)\}$ and hyponormal, if $T^*T - TT^* \geqq 0$. It is known that if T is hyponormal, then T is normaloid and $T - zI$ is also hyponormal for all complex numbers z.

When the original version of this paper was submitted, the referee told me of [3] then existing as a preprint and this makes possible the following shorter proof.

Proof of Theorem. Since $AN = N^*A$ and $0 \notin \mathrm{Cl}(W(A))$, $s(N)$ is real [3]. Also $\Sigma(N) = \mathrm{Cl}(W(N))$ for such a normaloid operator N [1]. Hence $\mathrm{Cl}(W(N))$ is real. This completes the proof of theorem.

The corresponding result for hyponormal operators now follows as corollary from this theorem and the remark made above.

The author expresses his thanks to Prof. U. N. Singh and the referee for their comments and suggestions.

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Received April 15, 1968.

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