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THE MAXIMAL SET OF CONSTANT WIDTH IN A LATTICE

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A new construction for sets of constant width is employed to determine the largest such set which will fit inside a square lattice.

A set W in E^2 is said to have *constant width* λ (denoted $\omega(W) = \lambda$) if the distance between each pair of parallel supporting lines of W is λ. If $x \in \mathrm{bd}\ W$ we will denote all points *opposite* x (that is, at a distance λ from x) in W by $0(x)$.

In what follows we will be most concerned with *Reuleaux polygons*, which are sets of constant width λ whose boundaries consist of an odd number of arcs of radius λ centered at other boundary points (see [2], p. 128, for a more complete description).

We say a set S *avoids* another set X if int $S \cap X = \varnothing$.

THEOREM 1. *Let L be a square planar unit lattice. Then the unique set of maximal constant width which avoids L is a Reuleaux triangle T having width $\omega(T) > 1.545$. An axis of symmetry of T parallels one of the major axex of L and is midway between two parallel rows of the lattice.*

The proof depends upon a variational method for altering Reuleaux polygons which will be described in § 2. A useful lemma is also proved there. In § 3 the proof of the theorem is given, while various generalizations are discussed in § 4.

The construction described in the next section was also found independently by Mr. Dale Peterson.

2. Variants of sets of constant width. Let P be a set of constant width λ and p_0 a point near P but exterior to it. Suppose that q and r are the two points on the boundary of P which are at a distance λ from p_0. Let Q be the convex set whose boundary is following: the shorter arc of the circle $C(p_0, \lambda)$ [the circle of radius λ centered at p_0] between q and r, the boundary of P from r to q' (a point opposite q), an arc of $C(q, \lambda)$ between q' and p_0, an arc of $C(r, \lambda)$ between p_0 and r', and the boundary of P from r' to q [see Figure 1]. We call Q the *p_0-variant* of P. It is easy to see that Q is a set of constant width λ. In order for the construction to work p_0 must be close enough to P so that the boundary arc of P between q and