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# AN INTERPOLATION PROBLEM FOR SUBALGEBRAS OF $H^\infty$

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**Let $E$ be a closed subset of the unit circle $C = \{z: |z| = 1\}$ and denote by $B_E$ the algebra of all functions which are bounded and continuous on the set $X = \{z: |z| \leqq 1, z \notin E\}$ and analytic in the open disc $D = \{z: |z| < 1\}$. An interpolation set for $B_E$ is a relatively closed subset $S$ of $X$ with the property that if $\alpha$ is a bounded and continuous function on $S$ (all functions are complex-valued), there is a function $f$ in $B_E$ such that $f(z) = \alpha(z)$ for every $z \in S$. The main result of the paper characterizes the interpolation sets for $B_E$ as those sets $S$ for which $S \cap D$ is an interpolation set for $H^\infty$ and $S \cap (C - E)$ has Lebesgue measure 0. If, in addition, $S \cap D = \phi$ then $S$ is a peak interpolation set for $B_E$. Also, through a construction process inspired by recent work of J. P. Kahane, it is shown that the existence of peak points for a sup norm algebra of continuous functions on a compact, connected space implies the existence of infinite interpolation sets relative to the algebra and certain of its weak extensions.**

The solution of the interpolation problem in the space $H^\infty = B_C$ of bounded analytic functions on $D$ is due to Lennart Carleson [5], and due to A. Beurling and Walter Rudin in the disc algebra $A = B_\phi$ [10]. Concerning the latter case see also the notes of Lennart Carleson [5] and the last problem in Hoffman's book [8]. Their results are given by the following two theorems.

THEOREM C. *A sequence $\{z_k\}$ of distinct points in $D$ is an interpolation set for $H^\infty$ if and only if it is uniformly separated*[1], *that is, if and only if there exists a positive number $\delta$ such that*

$$\prod_{j=1; j\neq k}^{\infty} \left| \frac{z_j - z_k}{1 - \bar{z}_j z_k} \right| \geqq \delta \qquad (k = 1, 2, \cdots) . \tag{1}$$

*Whenever this condition holds, a constant $m(\delta)$ exists with the property that for any bounded sequence $\{w_k\}$ there is an $f$ in $H^\infty$ such that $f(z_k) = w_k$ $(k = 1, 2, \cdots)$ and $\|f\| \leqq m(\delta) \sup_k |w_k|$.*

THEOREM B-R. *A closed subset $S$ of $\bar{D}$ is an interpolation set for $A$ if and only if*

(i) *$S \cap D$ is uniformly separated,*

[1] This terminology is due to Professor Peter Duren.