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A QUASI-DECOMPOSABLE ABELIAN GROUP WITHOUT PROPER ISOMORPHIC QUOTIENT GROUPS AND PROPER ISOMORPHIC SUBGROUPS

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All of the group in this paper are abelian p-groups without elements of infinite height. A group is said to be quasi-indecomposable if whenever H is a summand of G then either H or G/H is finite. The p-socle of G is the sub-group consisting of all the elements x in G such that $px = 0$.

In this paper it is shown that there are conditions that can be imposed on the socle of G which are sufficient for G to (a) have no proper isomorphic subgroups; (b) have no proper isomorphic quotient groups; and (c) be quasiindecomposable. Furthermore, it is shown that groups which make these results meaningful actually exist.

Let the cardinality of a group G be either $\aleph_0$ or greater than $c = 2^{\aleph_0}$. Then, as is well known, G has a proper isomorphic subgroup and a proper isomorphic quotient group. However P. Crawley [3] showed that the cardinality c is exceptional. He gave an example G_0 of cardinality c which has a standard basic subgroup and *no proper isomorphic subgroups.* After Crawley's example appeared, it was clear that a group, of cardinality c and with a standard basic subgroup, supplies examples of groups with strange but interesting properties. In fact R. S. Pierce [7] gave an example G_1 which has no proper isomorphic subgroups and *no proper isomorphic quotient groups.* And he gave also in [7] an example G_2 which is *quasi-indecomposable,* that is, every direct summand H of G_2 is either finite or G_2/H is finite.

The relationship between the above three properties (no proper isomorphic subgroups, no proper isomorphic quotient groups and quasi-indecomposability) of a group G with the cardinality c and a standard basic subgroup seems to authors an interesting problem. In this paper we shall give some results about this problem. In our approach the topological structure of the p-socle of the torsion completion of G will be used in an essential way. Theorem 1 tells us that the situation of the p-socle of G in the p-socle of the torsion completion of G gives us sufficient conditions for these three properties of G. In some sense it shows a relationship between the three properties. Theorem 2 shows the existence of a group which has all three properties. Theorem 3 shows the existence of a group which has no proper isomorphic subgroups and no proper isomorphic quotient groups but which is quasi-decomposable.

Now we want to add a simple proof of the following fact which