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SEMIVARIETIES AND SUBFUNCTORS OF THE IDENTITY FUNCTOR

S. A. HUQ

We study certain subcategories called semivarieties and obtain Kaplanasky's theorem on the decomposition of Abelian groups into a divisible group and a reduced group under the frame-work of category theory; We also investigate the connection of these epicoreflective subcategories with varieties.

Semivarieties are subcategories of a category with cartain axioms; such subcategories play an important role in Abelian categories and a description of these by means of coreflection, appears in the general context in the work of Mitchell [8, § 5, § 6, III]. Broader classes than these have also been studied by Amitsur [1], Carreau [2], under the title of $HI - RI$ properties of radicals and their classes. Our aim in this note is to give a categorical proof of Kaplanasky's Theorem 3 [6, § 5] and while so doing we generalize the concepts of varieties and variety functors of Fröhlich [4] under abstract frame work utilizing Maranda's [9] concept of a radical.

$\mathscr{C}$ is a category equipped with the following axioms:

I. $\mathscr{C}$ has a null object.

II. Every morphism α in $\mathscr{C}$, admits a factorization $\alpha = \nu\mu$, where ν is a normal epimorphism and μ is a monomorphism; we are writing the composition in the precise way $\underbrace{\cdot \xrightarrow{\alpha} \cdot \xrightarrow{\beta} \cdot}_{\alpha\beta}$ where the dots are the unnamed objects.

III. Every family of objects has a direct and a free product.

IV. The subobjects and factor objects of any objects form a set.

It is immediate that $\mathscr{C}$ admits null morphisms. The image of a morphism in $\mathscr{C}$, defined in axiom II and some time written as (ν, L, μ) in the factorization $\underbrace{\cdot \xrightarrow{\nu} L \xrightarrow{\mu} \cdot}_{\alpha}$, is uniquely determined to within equivalence. Every family of subobjects of an object A in $\mathscr{C}$ has a union and as such every morphism has a kernel. Dual consideration holds for factor objects and cokernel. A map admitting null subobject as the kernel is a monomorphism and a sequence

$$0 \longrightarrow A \xrightarrow{\alpha} B \xrightarrow{\beta} C \longrightarrow 0$$

is exact if and only if α is a monomorphism, β is a normal epimorphism and the subobject (A, α) serves as the kernel of β. For details on the notation used and results mentioned in this section the reader