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A MODULAR TOPOLOGICAL LATTICE

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The purpose of this paper is to present a construction of a compact connected topological lattice which is modular and not distributive. As a special case there will result the example which is a two dimensional subset of R^3, not embeddable in R^2.

The existence of such an example is related to structure questions in topological lattices considered by Dyer and Shields [3], Anderson [1], and others.

The first step is to present a general method for constructing a class of modular lattices. Let D denote a distributive lattice which is a chain, S a nonempty set, and L the S-fold product lattice of D. That is $L = \{f \mid f: S \rightarrow D\}$ and $f \leqq g$ if and only if $f(s) \leqq g(s)$ for every $s \in S$. It is known that $(L, \leqq)$ is a distributive lattice with its operations $\vee$ and $\wedge$ characterized by $[f \vee g](s) = f(s) \vee g(s)$ and

$$[f \wedge g](s) = f(s) \wedge g(s)$$

for every $s \in S$. Define

$M = \{f \in L \mid$ there exists $r \in S$ such that $s, t \in S - \{r\}$ implies

$$f(s) \leqq f(r) \quad \text{and} \quad f(s) = f(t)\} .$$

For intuition about M and the arguments that follow, note that M simply consists of all of the constant functions of L and the functions of L which are essentially constant in the sense that they assume but two values — the larger value at exactly one point.

If the order of L, $\leqq$, is restricted to M, it will be established through a sequence of lemmas that $(M, \leqq)$ is a modular lattice. Recall a lattice $(M, \vee, \wedge)$ is modular if and only if for every $a, b, c \in M$, $b \leqq a$ implies that $a \wedge (b \vee c) = b \vee (a \wedge c)$.

LEMMA 1. *If $f \in M$ and f is not constant, there exists a unique $r \in S$ such that $s, t \in S - \{r\}$ implies $f(s) < f(r)$ and $f(s) = f(t)$.*

The proof of the lemma is immediate from the definition of M, and consequently for $f \in M$ and not constant, define *index* f to be the unique element described in Lemma 1.

LEMMA 2. *$(M, \leqq)$ is a sub $\wedge$-semilattice of $(L, \leqq)$.*

It suffices to show that if $f, g \in M$, then $f \wedge g \in M$. If f and g are