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ON THE DECOMPOSITION OF INFINITELY DIVISIBLE CHARACTERISTIC FUNCTIONS WITH CONTINUOUS POISSON SPECTRUM, II

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Let f be an infinitely divisible characteristic function whose spectral functions are absolutely continuous functions with almost everywhere continuous derivatives. Some necessary conditions that f belong to the class I_0 of the infinitely divisible characteristic functions without indecomposable factors have been obtained by Cramér, Shimizu and the author and a sufficient condition that f belong to I_0 has been given by Ostrovskiy. In the present work, we prove that the condition of Ostrovskiy is not only a sufficient, but also a necessary condition that f belong to I_0.

Let f be the function of the variable t defined by

$$\begin{aligned}\log f(t) &= \int_{-\infty}^{0} [e^{itu} - 1 - itu(1+u^2)^{-1}]\varphi(u)du \\ &\quad + \int_{0}^{\infty} [e^{itu} - 1 - itu(1+u^2)^{-1}]\psi(u)du\end{aligned} \tag{1}$$

where log means the branch of logarithm defined by continuity from $\log f(0) = 0$ and where φ and ψ are almost everywhere nonnegative and continuous functions which are defined respectively on $]-\infty, 0[$ and $]0, +\infty[$ and satisfy the condition

$$\int_{-\varepsilon}^{0} u^2\varphi(u)du + \int_{0}^{\varepsilon} u^2\psi(u)du < +\infty$$

for any $\varepsilon > 0$. If we let

$$M(x) = \int_{-\infty}^{x} \varphi(u)du \qquad x < 0 ,$$

$$N(x) = -\int_{x}^{+\infty} \psi(u)du \qquad x > 0 ,$$

then we see that the conditions of the Lévy representation theorem ([4], Th. 5.5.2) are satisfied, so that f is an infinitely divisible characteristic function. In [3], we have proved the following result.

If the two following conditions are satisfied:

(a) $\varphi(u) \geqq k$ a.e. for $-c(1+2^{-n}) < u < -c$,

(b) $\psi(u) \geqq k$ a.e. for $d < u < d(1+2^{-n})$,

where k, c and d are positive constants and n is a positive integer,