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CHARACTERIZATION OF CERTAIN INVARIANT SUBSPACES OF H^p AND L^p SPACES DERIVED FROM LOGMODULAR ALGEBRAS

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Let $A = A(X)$ be a logmodular algebra and m a representing measure on X associated with a nontrivial Gleason part. For $1 \leqq p \leqq \infty$, let $H^p(dm)$ denote the closure of A in $L^p(dm)$ (w^* closure for $p = \infty$). A closed subspace M of $H^p(dm)$ or $L^p(dm)$ is called *invariant* if $f \in M$ and $g \in A$ imply that $fg \in M$. The main result of this paper is a characterization of the invariant subspaces which satisfy a weaker hypothesis than that required in the usual form of the generalized Beurling theorem, as given by Hoffman or Srinivasan.

For $1 \leqq p \leqq \infty$, let I^p be the subspace of functions in $H^p(dm)$ vanishing on the Gleason part of m and let $A_m = \left\{f \in A: \int f dm = 0\right\}$.

THEOREM. *Let M be a closed invariant subspace of $L^2(dm)$ such that the linear span of $A_m M$ is dense in M but the subspace $R = \{f \in M: f \perp I^\infty M\}$ is nontrivial and has the same support set E as M. Then M has the form $\chi_E \cdot F \cdot (\bar{I}^2)^\perp$ for some unimodular function F.*

A modified form of the result holds for $1 \leqq p \leqq \infty$. This theorem is applied to give a complete characterization of the invariant subspaces of $L^p(dm)$ when A is the standard algebra on the torus associated with a lexicographic ordering of the dual group and m is normalized Haar measure.

1. Invariant subspaces. In 1949 Beurling [1], using function analytic methods, showed that all the closed invariant subspaces of H^2 of the circle have the form $M = FH^2$, where $|F| \equiv 1$ a.e. In 1958 Helson and Lowdenslager [3] and [4] extended the result to some but not all subspaces of the H^2 space of the torus, using Hilbert space methods. In the past 10 years the latter arguments have been extended by Hoffman [5, Th. 5.5, p. 293], Srinivasan [8], [9], and others to prove the following generalized Beurling theorem. If m is a representing measure for a logmodular algebra A and if M is an invariant subspace of $L^2(dm)$ which is *simply* invariant, i.e., if

(1) the linear span of $A_m M$ is not dense in M,

then $M = FH^2$ for $|F| \equiv 1$. In the general case (even the torus case) not all invariant subspaces satisfy this hypothesis. Our purpose is to extend the characterization by weakening hypothesis (1).