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A SUBCOLLECTION OF ALGEBRAS IN A COLLECTION OF BANACH SPACES

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Let $D(p, r)$ with $1 \leqq p < \infty$ and $-\infty < r < +\infty$ denote the Banach space consisting of certain analytic functions $f(z)$ defined in the unit disk. A function $f(z) = \sum_{n=0}^{\infty} a_n z^n$ is a member of $D(p, r)$ if and only if

$$\sum_{n=0}^{\infty} (n+1)^r \mid a_n \mid^p < \infty .$$

We define the norm of f in $D(p, r)$ by

$$\| f \|_{p,r} = \left(\sum_{n=0}^{\infty} (n+1)^r \mid a_n \mid^p \right) 1/p .$$

By the product of two functions f and g in $D(p, r)$ we shall mean their product as functions, i.e., $[f. g](z) = f(z)g(z)$. The purpose of this paper is to discover which of the spaces $D(p, r)$ are algebras.

THEOREM 1. *If $D(p, r)$ is an algebra, then there exists a real $c > 0$ with $\| fg \| \leqq c \| f \| \| g \|$ for every $f, g \in D(p, r)$.*

Proof. Let h be a fixed element of $D(p, r)$. It suffices to show the map $f \rightarrow hf$ is a bounded linear transformation from $D(p, r)$ to itself. The proof is based on the closed graph theorem [2, p. 306]. Suppose h is a multiplier from $D(p_1, r_1)$ to $D(p_2, r_2)$ and suppose

(i) $f_n \rightarrow f$ in $D(p_1, r_1)$ and

(ii) $hf_n \rightarrow g$ in $D(p_2, r_2)$.

Then $f_n(z) \rightarrow f(z)$ for each z in the unit disk and so $h(z)f_n(z) \rightarrow h(z)f(z)$. On the other hand by (ii), $h(z)f_n(z) \rightarrow g(z)$ for each z in the unit disk. Hence $g = hf$, and so by the closed graph theorem multiplication by h is a continuous linear transformation. It follows from this [2, p. 183] that $D(p, r)$ is equivalent to a Banach algebra, and from this the theorem follows immediately.

COROLLARY 1. *If $D(p, r)$ is an algebra and $c > 0$ as above, then* $| f(z) | \leqq c \| f \| \ \forall f \in D(p, r)$ and $| z | < 1$.

Proof. For each f in $D(p, r)$ let T_f denote the multiplication operator from $D(p, r)$ to itself determined by f, i.e., $T_f(g) = fg$. Then for z_0 satisfying $| z_0 | < 1$ the map $T_f \rightarrow f(z_0)$ is a multiplicative linear functional on the Banach algebra of multiplication operators

$$T_f, f \in D(p, r)$$