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AN EMBEDDING THEOREM FOR LATTICE-ORDERED FIELDS

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In this paper we develop a method for constructing lattice-ordered fields ("$\mathscr{L}$-fields") which are not totally ordered ("*o*-fields") and hence are not *f*-rings. We show that many of these fields admit a Hahn type embedding into a field of formal power series with real coefficients. In order to establish such an embedding we make use of the valuation theory for abelian $\mathscr{L}$-groups and prove the "well known" fact that each *o*-field can be embedded in an *o*-field of formal power series.

Let G be an $\mathscr{L}$-field that contains n disjoint elements, but not $n+1$ such elements. An element $0 < s \in G$ is *special* if there is a unique $\mathscr{L}$-ideal of $(G, +)$ that is maximal without containing s. We show that the set S of special elements of G form a multiplicative group if and only if $S \neq \varnothing$ and $s^{-1} > 0$ for each $s \in S$. If this is the case, then there is a natural mapping of S onto the set Γ of all values of the elements of G. Thus Γ is a *po*-group and if, in addition, Γ is torsion free, then there exists an $\mathscr{L}$-isomorphism of G into the $\mathscr{L}$-field $V(\Gamma, R)$ of all functions v of Γ into the real field R whose support $\{\gamma \in \Gamma \mid v(\gamma) \neq 0\}$ satisfies the ascending chain condition. If G is an *o*-field, then the above hypotheses are satisfied and hence the embedding theorem for *o*-fields is a special case of our embedding theorem. The authors wish to thank the referee for many constructive suggestions.

NOTATION. If S is a subset of a group G, then $[S]$ will denote the subgroup of G that is generated by S. If G is a *po*-group, then G^+ will denote the set $\{g \in G \mid g \geqq 0\}$ of positive elements. A *disjoint* subset of an $\mathscr{L}$-group G is a set S of strictly positive elements such that $a \wedge b = 0$ for all pairs $a, b \in S$.

2. A method for constructing lattice-ordered rings.

A *po*-set Γ is called a *root system* if for each $\gamma \in \Gamma$, the set $\{\alpha \in \Gamma \mid \alpha \geqq \gamma\}$ is totally ordered. A nonvoid subset Δ of a root system Γ is called a *W-set* if it is the join of a finite number of inversely well ordered subsets of Γ, and an *I*-set if it is infinite and trivially ordered or well ordered with order type ω. In [2] it is shown that Δ is a W-set if and only if Δ does not contain an *I*-set; while in [10] five other conditions are derived which are equivalent to Δ not containing an *I*-set.