PACIFIC JOURNAL OF MATHEMATICS
Vol. 30, No. 3, 1969

# INTRINSIC EXTENSIONS OF RINGS

JOHN J. HUTCHINSON

**Faith posed the problem of characterizing the left intrinsic extensions of left quotient semisimple (simple) rings. In this paper a characterization is given for the left strongly intrinsic extensions of left quotient semisimple rings.**

Section 1 consists of several definitions and known preliminary results. In §2 we define essential subdirect sums and develop several of their elementary properties. The results of §2 enable us to state and prove the main characterization theorem which appears in §3. In the last section it is shown that in the class of left quotient semisimple rings, the left strongly intrinsic extensions are exactly the left intrinsic extensions.

**1. Preliminaries.** Let $R$ and $S$ be nonzero associative rings (not necessarily with identities or commutative) where $S \subseteqq R$. $S$ is *left quotient simple, left quotient semisimple,* a *left Ore domain* if $S$ has a left classical (and maximal) quotient ring which is respectively simple Artinian, semisimple Artinian, a division ring. The left classical quotient ring of $S$ will be denoted $\bar{S}$, and left quotient semisimple (left quotient simple) will be written lqss (lqs). $R$ is a *left intrinsic extension* of $S$ if every nonzero left ideal of $R$ has nonzero intersection with $S$. A left $S$-module $M$ (denoted ${}_SM$) is an *essential extension* of a submodule $N$ if every nonzero submodule of $M$ has nonzero intersection with $N$ (we also say $N$ is essential in $M$). $R$ is a *left essential extension* of $S$ if ${}_SR$ is an essential extension of ${}_SS$. It is clear that every left essential extension of $S$ is left intrinsic, but the converse is not always true (for instance when $R$ is a proper field extension of a field $S$). A left ideal $A$ of $S$ is *closed* if $S$ contains no proper left essential extensions of $A$ (as left $S$-modules). The symbol $L(S)$ will denote the set of closed left ideals of $S$. $R$ is a *left strongly intrinsic extension* of $S$ if $R$ is a left intrinsic extension of $S$, and for all $A \in L(S)$ there exists a left ideal $B$ of $R$ such that $B \cap S = A$. In any left $S$-module $M$, we denote by $Z({}_SM)$ the set of elements in $M$ whose annihilator in $S$ is an essential left ideal. Clearly $Z({}_SM)$ is a submodule of $M$.

THEOREM 1.1. *If $Z({}_SS) = 0$, then ${}_SS$ has a (unique up to isomorphism) maximal essential extension $Q$ (called the maximal quotient ring of $S$) which has a ring structure compatible with the module structure; and $Q$ is a regular, left self-injective ring such that*