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A NOTE ON p-SPACES AND MOORE SPACES

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In this note we investigate p-spaces and their relationship to Moore spaces. Specifically, it is shown that among p-spaces Moore spaces are equivalent to semi-metric spaces and spaces with a σ-discrete network. A certain class of p-spaces, called strict p-spaces, is given an internal characterization and this is used to show that a pointwise paracompact p-space with a point-countable base is a pointwise paracompact Moore space.

1. Developable p-spaces. Let us first discuss some of the definitions and basic concepts which will be used throughout this paper. Unless otherwise stated, all topological spaces are assumed to be T_2 and regular. The set of positive integers will be denoted by Z^+.

A sequence $\{\mathscr{U}_n\}_1^\infty$ of open covers of a space X is called a *development* for X if for any $x \in X$ and any open neighborhood O of x, there is an integer $n \in Z^+$ such that $St(x, \mathscr{U}_n) = \cup\{U \in \mathscr{U}_n: x \in U\} \subset O$. A regular developable space is a *Moore space*.

By Arhangel'skiî [1], a completely regular space X is called a *p-space* (plumed space) if in its Stone-Cech compactification $\beta(X)$ there is a sequence of families $\{\gamma_n\}_1^\infty$, where each γ_n is a collection of sets, open in $\beta(X)$, which covers X and satisfies: For each $x \in X$, $\bigcap_{n=1}^\infty St(x, \gamma_n) \subseteqq X$. The sequence $\{\gamma_n\}_1^\infty$ is called a *pluming* for X in $\beta(X)$. A space X is called a *strict p-space* if it has a pluming $\{\gamma_n\}_1^\infty$ with the following additional property: For any $x \in X$ and any $n \in Z^+$ there is $n(x) \in Z^+$ such that $\overline{St(x, \gamma_{n(x)})} \subseteqq St(x, \gamma_n)$. In this case we call $\{\gamma_n\}_1^\infty$ a *strict pluming*. The class of p-spaces includes all metric spaces, locally compact spaces and completely regular Moore spaces (see [1], [2]).

A collection $\mathscr{P}$ of subsets of a space X is called a *network* for X if for any open set $O \subseteqq X$ and $x \in O$ there is a set $P \in \mathscr{P}$ such that $x \in P \subseteqq O$.

Let X be a topological space and d a real valued nonnegative function defined on $X \times X$ which satisfies: For $x, y \in X$

(1) $d(x, y) = 0$ if and only if $x = y$;

(2) $d(x, y) = d(y, x)$.

The function d is called a *symmetric* [2] for the topology on X provided:

(3) $A \subseteqq X$ is closed in X if and only if $d(x, A) = \inf\{d(x, z): z \in A\} > 0$ for any $x \in X - A$.

The function d is called a *semi-metric* [9] for X provided:

(3′) For $A \subseteqq X$, $x \in \bar{A}$ if and only if $d(x, A) = 0$.

It is clear that a semi-metric space is a symmetric space and it can