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THE BENDING OF SPACE CURVES INTO PIECEWISE HELICAL CURVES

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It is the purpose of this paper to show that a regular C^3 space curve Γ in a Euclidean 3-space, whose curvature $\kappa \neq 0$, can be bent into a piecewise helix (i.e., a curve that is a helix but for a finite number of corners) in such a way that the piecewise helix remains within a tubular region about C of arbitrarily small preassigned radius. Moreover, we shall show that the bending can be carried out in such a way that either (a) the piecewise helix is circular or (b) the piecewise helix has the same curvature as Γ at corresponding points except possibly at corners, of (c) if the torsion of Γ is nowhere zero, then the piecewise helix has the same torsion as Γ at corresponding points except possibly at corners.

Also we shall show that if, in addition, Γ has a bounded fourth derivative, then an explicit formula can be given for a sufficient number n of helices that make up the piecewise helix, where n depends on Γ and the radius of the tubular region about Γ. In this case, we shall also show how the determination of the piecewise helix can be reduced to a problem in simple integration.

1. Bendability.

DEFINITION 1. A curve is called a *piecewise helix* if it consists of a finite number of segments, each of which is a helix (i.e., a curve whose tangent makes a constant angle with a fixed direction). A point at which two consecutive helices meet will be called a *corner* of the piecewise helix.

REMARK. If, in particular, between corners the helix is a circular helix, then the piecewise helix will be called a *piecewise circular helix.*

THEOREM 1. *Let Γ: $r(s)$, s = arc length, $0 \leqq s \leqq l$, be a regular $C^3[0, l]$*[1] *curve whose curvature $\kappa(s)$ is nowhere zero. Then for any given $\varepsilon > 0$*

(a) *there exists a piecewise circular helix Γ_1^*: $h_1^*(s)$, s = arc length, $0 \leqq s \leqq l$, such that:*

$$|r(s) - h_1^*(s)| < \varepsilon\,, \qquad 0 \leqq s \leqq l\,;$$

[1] (I.e., $r(s)$ can be extended to lie in C^3 on some open set containing $0 \leqq s \leqq l$.)