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THE ADJOINT GROUP OF LIE GROUPS

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Let G be a Lie group and let Aut(G) denote the group of automorphisms of G. If the subgroup Int(G) of inner-automorphisms of G is closed in Aut(G), then we call G a (CA) group (after Van Est.). In this note, we investigate (CA) property of certain classes of Lie groups. The main results are as follows:

THEOREM A. **Let G be an analytic group and suppose that there is no compact semisimple normal subgroup of G. If G contains a closed uniform (CA) subgroup H, then G is (CA).**

THEOREM B. **If G is an analytic group whose exponential map is surjective, then G is (CA).**

In [3], Garland and Goto proved that if an analytic group G contains a lattice, then G is (CA). Since a lattice in a solvable group is a uniform lattice, it is finitely generated and so the automorphism group of this uniform lattice is discrete, and thus this lattice is trivially a (CA) subgroup. Thus Theorem A generalizes the above theorem of Garland and Goto for solvable groups. Theorem B is an improvement of the well known theorem that every nilpotent analytic group is (CA) (see [2]). In §1, we introduce some notation and preliminary materials. §2 and §3 are devoted for the proofs of the main theorems together with their immediate corollaries.

1. Preliminaries and notations. The group Aut(G) of automorphisms of locally compact a topological group G may be regarded as a topological group, the topology being the (generalized) compact open topology defined as in [5]. Thus, if we denote by $N(C, V)$ the set of all $\theta \in \mathrm{Aut}(G)$ for which $\theta(x)x^{-1} \in V$ and $\theta^{-1}(x)x^{-1} \in V$ whenever $x \in C$, then the sets $N(C, V)$ form a fundamental system of neighborhoods of the identity element of Aut(G) as C ranges over the compact subsets of G and V over the set of neighborhoods of the identity element of G.

If G is an analytic group and $\mathscr{G}$ its Lie algebra, then Aut(G) may be identified with a closed subgroup of the linear group Aut($\mathscr{G}$) of automorphisms of $\mathscr{G}$. Under this identification, Int(G) coincides with the adjoint group Int($\mathscr{G}$), which is generated by e^{adX}, $X \in \mathscr{G}$ where ad denotes the adjoint representation of $\mathscr{G}$. Thus the (CA) property of analytic groups are entirely determined by their Lie algebras. In particular, if $\tilde{G}$ is a covering group of G and if G is (CA), then so is $\tilde{G}$. This fact is used in the proofs of the main theorems.