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A CLASSIFICATION OF CENTER-FOCI

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The main purpose of this paper is to classify center-foci according to isomorphisms. Necessary and sufficient conditions are found for an isomorphism to exist in terms of properties on the cycles in suitable neighborhoods of the center-foci. In the last section $o+$ stable isolated critical points are classified according to isomorphisms.

This paper was motivated by discussions with Otomar Hájek and Taro Ura.

Throughout this paper R^1, R^+, R^-, and R^2 will denote the reals, the nonnegative reals, the nonpositive reals, and the plane respectively.

Let F be a family of curves filling a region R of the plane. F will be said to be regular at a point p of R if F is locally homeomorphic with parallel lines at p. F is called regular in R if it is regular at each point of R. A cross-section of F (through the point p of R) shall mean an arc T (of which p is a nonend-point) which lies in R and is such that each curve of F intersects T at most once.

Let (X, π) be a dynamical system on X, i.e., X is a topological space and π is a mapping of $X \times R^1$ onto X satisfying the following axioms: (where $x\pi t = \pi(x, t)$ for $(x, t) \in X \times R^1$)

(1) $x\pi 0 = x$ for $x \in X$

(2) $(x\pi t)\pi s = x\pi(t + s)$ for $x \in X$ and $t, s \in R^1$

(3) π is continuous.

For $x \in X$, $x\pi R^1$ is called the trajectory through x and is denoted by $C(x)$. If $C(x) = \{x\}$, x is called a critical point. If there exists $t \in R^1$, $t \neq 0$, such that $x\pi t = x$, then x is called a periodic point. If x is a periodic point and not a critical point, $C(x)$ is called a cycle. A subset A of X is said to be invariant if $A\pi R^1 = A$, or $+$ invariant if $A\pi R^+ = A$. A subset B of X is said to be

(1) orbitally stable (o stable) if B has arbitrarily small invariant neighborhoods.

(2) orbitally $+$ stable ($o+$ stable) if B has arbitrarily small $+$ invariant neighborhoods.

(3) asymptotically orbitally $+$ stable ($ao+$ stable) if B is $o+$ stable and for some neighborhood U of x, $L^+(x) \subset \bar{B}$ for every $x \in U$, where $L^+(x)$ is the positive limit set of x.

Let (X, π) be a dynamical system on a metric space X. The positive prolongational limit set, denoted by $J^+(x)$, of a point $x \in X$ is given by $J^+(x) = \{y$: there exist sequences $\{x_i\} \subset X$ and $\{t_i\} \subset R^1$ such that $x_i \to x$, $t_i \to +\infty$, and $x_i\pi t_i \to y\}$. The negative prolongational limit