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HOMOTOPY GROUPS OF PL-EMBEDDING SPACES

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Let N be a compact PL-n-manifold, and let M be a PL-m-manifold without boundary. Two of the major problems in PL-topology are to determine conditions such that (1) any continuous map of N into M can be homotoped to a PL-embedding, and (2) two homotopic PL-embeddings are PL-isotopic.

If $C(N, M)$ is the space of continuous maps of N into M with the compact open topology, and if $PL(N, M)$ is the subspace of PL-embeddings, one can consider the map $i_\#: \Pi_0(PL(N, M)) \rightarrow \Pi_0(C(N, M))$ induced by inclusion. If (1) is true, then $i_\#$ is onto; if (2) is true, then $i_\#$ is one-to-one. In this paper, we investigate the higher homotopy groups of $PL(N, M)$ and $C(N, M)$.

Irwin has shown that if N is a closed manifold, $m \geqq n + 3$, then sufficient conditions for (1) are that N is $(2n - m)$—connected and M is $(2n - m + 1)$—connected. By raising the connectivities of N and M by one, Zeeman [7] proved (2).

By using Proposition 1 of Morlet [4] and Irwin [3], one can easily show the following theorem by using techniques similar to the proof of Theorem 2 below.

THEOREM 1. *Let N be a closed $(2n + s + 1 - m)$—connected PL-n-manifold and let M be a $(2n + s + 2 - m)$—connected PL-m-manifold without boundary, $m \geqq n + 3$. The homomorphism $i_\#$: $\Pi_s(PL(N, M)) \rightarrow \Pi_s(C(N, M))$ induced by inclusion is an isomorphism; if the connectivities of N and M are lowered by one, then $i_\#$ is onto.*

An analogous theorem in the differential case has been proved by J. P. Dax [1], [2].

If N has a nonempty boundary, then Dancis, Hudson and Tindell (independently and unpublished) have shown that if N has a k-dimensional spine with $m \geqq \{n + 3,\ n + k\}$, this is a sufficient condition for (1). If $m \geqq \{n + 3,\ n + k + 1\}$, they obtain (2). We generalize.

THEOREM 2. *Let N be a compact PL-n-manifold with k-spine K, $k < n$, and let M be a PL-m-manifold without boundary. If $m \geqq n + k + s + 1$, the homomorphism $i_\#$: $\Pi_s(PL(N, M)) \rightarrow \Pi_s(C(N, M))$ induced by inclusion is an isomorphism; if $m \geqq n + k + s$, $i_\#$ is onto.*

Note that the codimension 3 restriction is eliminated. In § 3,