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MESOCOMPACTNESS AND RELATED PROPERTIES

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This paper is concerned with some of those generalizations of paracompactness which can arise by broadening the concept of local finiteness, e.g., metacompactness, in contrast to those which come about by varying the power of an open cover, e.g., countable paracompactness. Quite recently, several generalizations of the first type have been studied. These include mesocompactness and sequential mesocompactness, strong and weak cover compactness, and Property Q.

In § 1, the notion of metacompactness (=pointwise paracompactness) is used to establish a hierarchy among these concepts, and in regular r-spaces, some of these notions are shown to be equivalent to paracompactness. In § 2, it is shown that mesocompactness is an invariant, in both directions, of perfect maps and that unlike paracompact spaces, there exists a mesocompact T_3 space which is not normal, and a mesocompact T_2 space which is not regular.

Throughout this paper, a space will mean a *Hausdorff space*. A convergent sequence in a space will mean the sequence and its limit, and we will use $\mathrm{Cl}(A)$ to denote the closure of A.

1. Related properties. We will begin with some pertinent definitions.

DEFINITION 1.1. A family $\mathscr{F}$ of sets in a space X is called *compact-finite* (*convergent sequence finite*) if every compact set (convergent sequence) in X meets at most finitely many members of $\mathscr{F}$. A space X is called *mesocompact* (*sequentially mesocompact*) if every open cover of X has a compact-finite (convergent sequence finite) open refinement (see [3]). We will use the abbreviation cs-finite for convergent sequence finite.

DEFINITION 1.2. A cover $\mathscr{V}$ of a space X is called *strong cover compact* if whenever $\{V_i; i \in N\}$ is a countably infinite subcollection of distinct elements of $\mathscr{V}$, p_i and $q_i \in V_i$ for each i, with $p_i \neq p_j$ and $q_i \neq q_j$ for $i \neq j$ and the point set $\{p_i;\ i \in N\}$ has a limit point in X, then the point set $\{q_i;\ i \in N\}$ has a limit point in X.

DEFINITION 1.3. A space X is *strong cover compact* if every open cover of X has a strong cover compact open refinement. We will use the abbreviation scc for strong cover compact.