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WILD POINTS OF CELLULAR ARCS IN 2-COMPLEXES IN E^3 AND CELLULAR HULLS

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Loveland has established that if W is the set of wild points of a cellular arc that lies on a 2-sphere in E^3, then either W is empty, W is degenerate, or W contains an arc. This note considers 2-complexes rather than 2-spheres. Making strong use of Loveland's results and others, it is proved that a cellular arc in a 2-complex in E^3 either contains an arc of wild points or has at most one wild point that has a neighborhood in the 2-complex homeomorphic to an open 2-cell. In the case of noncellular arcs in E^3, one can investigate "minimal cellular sets" containing the arc. A *cellular hull* of a subset A of E^3 is a cellular set containing A such that no proper cellular set also contains A. A characterization is given of those arcs in E^3 that have cellular hulls that lie in tame 2-complexes in E^3.

A *2-complex* in E^3 is the homeomorphic image of a 2-dimensional finite Euclidean polyhedron. A subset X of E^3 is said to be *locally tame* at a point p of X if there is a neighborhood N of p in E^3 and a homeomorphism h of $\mathrm{Cl}(N)$ (Cl = closure) onto a polyhedron in E^3 such that $h(\mathrm{Cl}(N \cap X))$ is a finite Euclidean polyhedron. A point p of a subset X of E^3 is said to be a *wild point* of X if X is not locally tame at p. A subset G of E^3 is said to be *cellular* (in E^3) if there exists a sequence $Q_1, Q_2, \cdots$ of 3-cells in E^3 such that for each positive integer i, $Q_{i+1} \subset \text{Interior } Q_i$ and $G = \bigcap_{i=1}^{\infty} Q_i$. If A and B are two arcs in E^3, then A is said to be *equivalent* to B if there is a homeomorphism h mapping E^3 onto E^3 such that $h(A) = B$.

THEOREM 1. *Let A be a cellular arc in a 2-complex in E^3. If the set of wild points of A does not contain an arc, then A has at most one wild point that has a neighborhood in the 2-complex homeomorphic to an open 2-cell.*

Proof. Assume that A has two wild points p and q that have neighborhoods in the 2-complex homeomorphic to an open 2-cell and contradict the hypothesis that A is cellular. Then p lies on a subarc of A that is contained in the interior of a closed 2-cell. The argument of Theorem 5 of [3] then establishes that p lies on a subarc C of A that is contained in a 2-sphere in E^3. Since C is a cellular arc by [6], it follows from [5] that p is the only wild point of C. Thus p and q are isolated wild points of A.