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# ON UNIFORM CONVERGENCE FOR WALSH-FOURIER SERIES

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**In 1940 R. Salem formulated a sufficient condition for a continuous and periodic function to have a trigonometric Fourier series which converges uniformly to the function. In this paper we will formulate a similar condition, which implies that the Walsh-Fourier series of such a function has this property. Furthermore we show that our result is stronger than certain classical results, and that it also implies the uniform convergence of the Walsh-Fourier series of certain classes of continuous functions of generalized bounded variation. The latter is analogous to results obtained by L. C. Young and R. Salem for trigonometric Fourier series.**

Let $\{\varphi_n(x)\}$ be the sequence of Rademacher functions, i.e.,

$$\varphi_0(x) = +1 \left(0 \leqq x < \frac{1}{2}\right), \qquad \varphi_0(x) = -1 \left(\frac{1}{2} \leqq x < 1\right),$$
$$\varphi_0(x+1) = \varphi_0(x) .$$

$\varphi_n(x) = \varphi_0(2^n x)$, $(n = 1, 2, 3, \cdots)$. In [3] R. E. A. C. Paley gave the following definition for the Walsh functions $\{\psi_n(x)\}$: $\psi_0(x) \equiv 1$, and, if $n = 2^{n_1} + 2^{n_2} + \cdots + 2^{n_r}$, with $n_1 > n_2 > \cdots > n_r$, then $\psi_n(x) = \varphi_{n_1}(x)\varphi_{n_2}(x) \cdots \varphi_{n_r}(x)$. J. L. Walsh [6] proved that the system $\{\psi_n(x)\}$ is a complete orthonormal system. For every Lebesgue-integrable function $f(x)$ of period 1 there is a corresponding Walsh-Fourier series (WFS):

$$f(x) \sim \sum_{k=0}^{\infty} c_k \psi_k(x), \quad \text{with} \quad c_k = \int_0^1 f(t)\psi_k(t)dt .$$

As in the case of trigonometric Fourier series (TFS), we can find a simple expression for the partial sums of a WFS,

$$S_n(f, x) = \sum_{k=0}^{n-1} c_k \psi_k(x) = \int_0^1 f(x \dotplus t) D_n(t) dt ,$$

where $D_n(t) = \sum_{k=0}^{n-1} \psi_k(t)$. For the meaning of $\dotplus$ and for further notations, definitions and properties of the WFS we refer to [2].

2. In [4], Chapter VI, R. Salem proved the following theorem: Let $f(x)$ be a continuous function of period $2\pi$. For odd $n$, let

$$T_n(x) = \sum_{p=0}^{(n-1)/2} (p+1)^{-1}[f(x + 2p\pi/n) - f(x + (2p+1)\pi/n)]$$