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REGULAR-CLOSED SPACES AND PROXIMITIES

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The theory of the compactifications of a completely regular space has been elucidated in recent years by the theory of proximities, introduced by Efremovič and developed especially by Smirnov. The two fundamental results are that a space has a compactification if and only if it has the topology of some proximity, and that there is a one-to-one correspondence from the collection of compactifications of a space onto the collection of proximities that give the topology of the space. We shall generalize these results to a larger class of spaces, which are related to the regular-closed spaces in the same manner as completely regular spaces are related to compact spaces.

We recall that a space is said to be *regular-closed* if it is regular, and cannot be nontrivially densely embedded in a regular space. Since every compact space is regular-closed, then any completely regular space can be embedded in a regular-closed space, namely any compactification of it. It has been shown by Berri and Sorgenfrey [1] that a regular-closed space need not be compact, and it has been shown by Herrlich [4] that there is a regular space that cannot be densely embedded in a regular-closed space. It follows from these remarks that the class of spaces which can be densely embedded in a regular-closed space, which we call the class of *RC-regular* spaces, lies properly between the class of regular spaces and the class of completely regular spaces.

The preceding remarks lead us to ask for a characterization of those spaces that can be densely embedded in a regular-closed space. We provide such a characterization in terms of a generalization of the theory of proximities to the theory of RC-proximities, and we also establish a one-to-one onto correspondence between regular-closed embeddings and RC-proximities for an RC-regular space.

The term regular as used herein includes T_1 separation.

2. Proximities.

We introduce axioms describing a relation between subsets of a space, which we shall call a proximity relation. These axioms are those of the usual theory of proximities, which we shall call the theory of *completely regular proximities*, with the exception of the axiom of complete regularity, which we do not use.

A *proximity* on X is a symmetric relation δ between subsets of X satisfying the following four conditions: