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RANK PRESERVERS OF SKEW-SYMMETRIC MATRICES

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It is possible to study the structure of rank preservers on n-square skew-symmetric matrices over an algebraically closed field F by considering instead the linear transformations on the second Grassmann Product Space $\wedge^2\mathscr{U}$ ($\mathscr{U}$ an n-dimensional vector space) over F into itself, which preserve the irreducible lengths of the products. In this paper, it is shown that preservers of irreducible length 2 are also preservers of all irreducible lengths of the products. Correspondingly, rank 4 preservers are rank $2k$ preservers for all positive integer values of k. The structure of the preservers in each case is deduced from the fact that these preservers are in particular irreducible length 1 and rank 2 preservers respectively, whose structures are known.

A nonzero vector in $\wedge^2\mathscr{U}$ is said to have *irreducible length k* if it can be written as a sum of k and *not less than k* pure (decomposable) nonzero products in $\wedge^2\mathscr{U}$. The set of such products is denoted by $\mathscr{L}_k$ and $z \in \mathscr{L}_k$ if and only if $\mathscr{L}(z) = k$. A linear transformation $\mathscr{T}$ of $\wedge^2\mathscr{U}$ into itself is an *$\mathscr{L}$-k preserver* if and only if $\mathscr{T}(\mathscr{L}_k) \subseteqq \mathscr{L}_k$.

A linear transformation $\mathscr{S}$ which takes the set of *rank 2k n*-square skew-symmetric matrices into itself is a *ρ-2k preserver*.

In [7], it is shown that $\mathscr{L}_k$ is isomorphic to the set of all rank $2k$ n-square skew-symmetric matrices. If this isomorphism is denoted by φ, then $\mathscr{S} = \varphi\mathscr{T}\varphi^{-1}$ is a ρ-$2k$ preserver if and only if $\mathscr{T}$ is a $\mathscr{L}$-k preserver.

To obtain the results of this paper, much use is made of $\mathscr{L}$-2 subspaces of $\wedge^2\mathscr{U}$. An *$\mathscr{L}$-k subspace* of $\wedge^2\mathscr{U}$ is a vector subspace whose nonzero members are in $\mathscr{L}_k$. An $\mathscr{L}$-2 subspace H is called a *(1, 1)-type subspace* if there exist fixed nonzero vectors $x \neq y$ such that each nonzero $f \in H$ can be written

$$f = x \wedge x_f + y \wedge y_f .$$

1. Intersection of (1, 1)-type subspaces.

LEMMA 1. *If V_1, V_2 are distinct (1, 1)-type subspaces of dimension $\geqq 2$ and $\dim V_1 \cap V_2 \geqq 2$, then the 2-dimensional subspaces of $\mathscr{U}$ determined by V_1, V_2 are equal.*

Proof. Let f_1, f_2 be independent in $V_1 \cap V_2$. Then $f_1 = x \wedge x_1 + y \wedge y_1$,