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ON A CERTAIN GENERALIZATION OF ℓ_p SPACES

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An $\mathscr{E}_p$ space is a product of finite-dimensional c_p spaces with a weighted ℓ_p norm on the product. The first theorem of this paper yields an isometric embedding of $\mathscr{E}_p$ into an appropriate c_p space. From this theorem, known results about c_p are used to deduce, among other things, the Clarkson inequalities for $\mathscr{E}_p$, $1 < p < \infty$, and hence, the uniform convexity of $\mathscr{E}_p$ for $1 < p < \infty$.

The second theorem characterizes the conjugate space of $\mathscr{E}_p$ for $0 < p < 1$. This result is then used to describe some spaces of multipliers. Let $\mathscr{A}$ and $\mathscr{B}$ be $\mathscr{E}_p$ spaces, $1 \leqq p \leqq \infty$, or $\mathscr{E}_0$. The spaces $\mathscr{M}(\mathscr{A}, \mathscr{B})$ of multipliers from $\mathscr{A}$ to $\mathscr{B}$ have previously been identified with certain subspaces of $\mathscr{E}(I)$ and determined precisely in some cases. The third theorem is a complete description of these multiplier spaces: the cases $0 < p < 1$ are included and the spaces $\mathscr{M}(\mathscr{A}, \mathscr{B})$ are determined precisely for all pairs $\mathscr{A}, \mathscr{B}$.

1. Definitions. First, we repeat the definition of c_p (called C_p by Dunford and Schwartz [1], S_p by Gohberg and Krein [2], and c_p by McCarthy [6]). See also [3, D. 37] for the case where H is finite-dimensional.

DEFINITION 1.1. Let H be a Hilbert space and let X be a compact operator on H. Then XX^* is positive and compact and hence has a unique positive square root which is also compact. We denote this square root by $|X|$. Now let μ_n be the, at most countably many, nonzero eigenvalues of $|X|$ enumerated with their multiplicity and arranged in a decreasing sequence as $\mu_1 \geqq \mu_2 \geqq \cdots \geqq 0$. For $0 < p < \infty$, we define

$$||X||_{\phi_p} = \left(\sum_{n=1}^{\infty} \mu_n^p\right)^{1/p}$$

whether finite or infinite; and we define

$$||X||_{\phi_\infty} = \sup\{\mu_n : 1 \leqq n < \infty\} = \mu_1 .$$

Equivalently, [1, p. 1089], $||X||_{\phi_\infty}$ is the operator norm of X. Then c_p consists of all compact X with $||X||_{\phi_p}$ finite.

See [1], [2], and [6] for a detailed treatment of c_p spaces and for additional references. Also, [3, Appendix D] contains a number of results in case H is finite-dimensional.