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DECOMPOSABLE SYMMETRIC TENSORS

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A *k-field* is a field over which every polynomial of degree less than or equal to k splits completely. The main theorem characterizes the maximal decomposable subspaces of the k^{th} symmetric space $\bigvee_k V$, where V is finite-dimensional vector space over an infinite k-field. They come in three forms:

(1) $\{x_1 \vee \cdots \vee x_k : x_k \in V\}$, $x_1, \cdots, x_{k-1}$ **fixed;**

(2) $\langle a, b\rangle_k = \{x_1 \vee \cdots \vee x_k : x_i \in \langle a, b\rangle\}$; **and**

(3) $\{x_1 \vee \cdots \vee x_{k-r} \vee \langle a, b\rangle_{(r')}\}$, $x_1, \cdots, x_{k-r}$ **fixed;**

where a and b are linearly independent vectors in V and $\langle a, b\rangle$ is the subspace spanned by a and b.

We consider symmetric tensor products of vector spaces and the problem of characterizing their maximal decomposable subspaces. This problem has been resolved in the skew-symmetric case by Westwick [4] using results due to Wei-Liang Chow [1, Lemma 5] when the underlying field is algebraically closed with characteristic zero.

A *k-field* is a field F over which every polynomial of degree at most k splits completely. In this paper we determine the maximal decomposable subspaces in the symmetric case when the underlying vector space is finite-dimensional over an infinite k-field whose characteristic (if any) exceeds the length of the product.

1. Let F be a field and V a vector space over F. The k-fold Cartesian product of V will be denoted by V^k where $1 < k$. A *rank k symmetric tensor space* is a vector space together with a k-multilinear symmetric mapping σ which is universal for k-multilinear symmetric maps of V^k and is spanned by $\sigma(V^k)$. We will use the notation $\bigvee_k V$ for this space. (The anti-symmetric or Grassman space is usually denoted by $\bigwedge^k V$.)

If $\bigvee_k V$ with σ: $V^k \to \bigvee_k V$ is a symmetric tensor space, the *decomposable symmetric tensors* or "symmetric products" are those elements of $\bigvee_k V$ in the set $\sigma(V^k)$. We will denote $\sigma(x_1, \cdots, x_k)$ by $x_1 \vee \cdots \vee x_k$. A subspace S of $\bigvee_k V$ is decomposable if $S \subseteqq \sigma(V^k)$. *Trivial decomposable subspaces* are the zero subspace and those consisting of scalar multiples of a single product. The *factors* of the product $x_1 \vee \cdots \vee x_k$ are the 1-dimensional subspaces $\langle x_1\rangle, \cdots, \langle x_k\rangle$ of V.

If V is n-dimensional, it is well-known that $\bigvee_k V$ is vector space isomorphic to the space of homogeneous polynomials of degree k over F [3, p. 428]. Any linear mapping f: $V \to V$ induces a unique linear mapping $\bigvee_k f$: $\bigvee_k V \to \bigvee_k V$ obtained by extending the mapping