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ON NETS OF CONTRACTIVE MAPS IN UNIFORM SPACES

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R. B. Fraser and S. Nadler have recently proved the following theorem: If X is a locally compact metric space, if $f_n \to f_0$ pointwise, where each f_n, $n = 0, 1, 2, \cdots$ is a contractive map with fixed point a_n, then $f_n \to f_0$ uniformly on compacta, and $a_n \to a_0$. Their method of proof actually showed more. In fact it implied that if a_0 was a fixed point of f_0, and if U is a compact neighborhood of a_0, then there exists a natural number $N(U)$ such that if $n \geqq N(U)$ then f_n had a fixed point $a_n \in U$, and $a_n \to a_0$. In 1963, W. J. Kammerer and R. H. Kasriel proved a theorem giving conditions for existence and uniqueness of fixed points of a general type contractive map on a uniform space. Edelstein in 1965, was able to considerably strengthen their results and achieved a significant extension of the Banach fixed point theorem. In this paper we show that the theorem of Fraser and Nadler may be extended with minor alteration to include locally compact uniform spaces. It was evident in the context of uniform spaces that the covergent sequences of their theorem should be replaced by covergent nets. Our method of proof is similar to their proof and used Edelstein's fixed point theorem.

1. Some preliminary results.

In what follows let $(X, \mathfrak{U})$ be a uniform space and let f be a mapping of X into itself. Let $\mathfrak{B}$ be a symmetric base for the uniformity $\mathfrak{U}$.

DEFINITION A. f is said to be a $\mathfrak{B}$-contraction if for each $U \in \mathfrak{B}$ and $(x, y) \in U$, $x \neq y$, a $W \in \mathfrak{B}$ exists such that $(f(x), f(y)) \in W \subset \operatorname{int} U$, and $(x, y) \notin W$.

DEFINITION B. f is said to be $\mathfrak{B}$-contractive if for each $U \in \mathfrak{B}$ and $(x, y) \in U$, $x \neq y$, a $W \in \mathfrak{B}$ exists such that $(f(x), f(y)) \in W \subset U$ and $(x, y) \notin W$.

DEFINITION C. $\mathfrak{B}$ is said to be ample if whenever $(x, y) \in U \in \mathfrak{B}$ there is $V \in \mathfrak{B}$ such that $(x, y) \in V \subset \bar{V} \subset U$.

REMARK. Except for minor changes the above terminology agrees with that used in the papers of Rhodes [8], Brown and Comfort [1], Kamerer and Kasriel [5], Edelstein [3], and Knill [7]. In addition, throughout we will use notation as standard in Kelley [6].