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COVERINGS OF PRO-AFFINE ALGEBRAIC GROUPS

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The author has shown [Illinois J. Math. (1969)] that a connected affine algebraic group over an algebraically closed field of characteristic 0 has a universal affine covering if and only if its radical is unipotent. The attempt to construct universal coverings of arbitrary connected affine algebraic groups forces the acceptance of pro-affine algebraic groups, whose Hopf algebras of polynomial functions are not necessarily finitely generated. This is a motivation for extending the covering theory over the larger category of pro-affine algebraic groups.

As we shall see here, the basic methods and results from the theory of affine algebraic groups extend easily and smoothly so as to yield the appropriate results concerning coverings of pro-affine algebraic groups over an algebraically closed field of characteristic 0. In particular, the Lie algebras of these groups can be used for obtaining a simple and natural construction of universal coverings.

Our reconsideration of the covering theory also fills a gap in the theory for affine groups. One of our main results gives a characterization, in algebraic-geometric terms, of those 'space coverings' which arise from group coverings. The simplicity of this characterization is undoubtedly due to the assumption that the base field be of characteristic 0.

For the basic elementary notions and results concerning pro-affine algebraic groups and their Hopf algebras of polynomial functions, we refer the reader to [2]. A few general features of this context that are not covered in [2] are dealt with in § 2. The initial results concerning coverings, valid also for base fields of non-zero characteristic, are contained in § 3. The main results, most of which require that the base field be of characteristic 0, are contained in §'s 4 and 5.

2. Some basic generalities. The structure of a pro-affine algebraic group over a field F consists of a group G, together with a Hopf algebra $A = \mathscr{A}(G)$ of F-valued functions on G, which are called the polynomial functions. These polynomial functions are representative functions, in the sense that, for each polynomial function f, all the translates $x\cdot f\cdot y$, with x and y in G, lie in some finite-dimensional F-space of functions. Here, $x\cdot f\cdot y$ is defined by $(x\cdot f\cdot y)(z) = f(yzx)$. It is assumed that A separates the elements of G, so that we may view the elements of G as F-algebra homo-