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A v-INTEGRAL REPRESENTATION FOR LINEAR OPERATORS ON SPACES OF CONTINUOUS FUNCTIONS WITH VALUES IN TOPOLOGICAL VECTOR SPACES

J. R. EDWARDS AND S. G. WAYMENT

Suppose X and Y are topological vector spaces. This paper gives an analytic representation of continuous linear operators from C into Y, where C denotes the space of continuous functions from the interval $[0, 1]$ into X with the topology of uniform convergence. In order to obtain an integral representation theorem analogous to the ones given by R. K. Goodrich for the locally convex setting in Trans. Amer. Math. Soc. 131 (1968), 246–258, certain strong hypotheses on C must be assumed because of the need to be able to extend the operators to a subset of the double dual of C. However, by using the notion of v-integral, it is possible to avoid this problem and give a representation theorem without additional hypothesis.

Let $\mathscr{I}$ be the collection of intervals in $(0, 1]$ of the form $(a, b]$ and let $L[X, Y]$ denote the space of linear operators from X into Y. Then the set function K from $\mathscr{I}$ into $L[X, Y]$ is said to be convex with respect to length if $K(I) = \sum_{i=1}^{n} [l(I_i)/l(I)]K(I_i)$ whenever $I = \bigcup_{i=1}^{n} I_i$, and where $1(I)$ denotes the length of I. If K is convex with respect to length, then K is said to be v-integrable with respect to f if $\lim_{|\sigma|\to 0} \sum K(I_i)(\varDelta_i f) = v\int K df$ exists in $\bar{Y}$, the completion of Y (by $\varDelta_i f$ we mean $f(t_{i+1}) - f(t_i)$ where $\{t_i\}$ is the partition σ of $[0, 1]$).

If $I \in \mathscr{I}$, with endpoints a and b, then the function $\varPsi_I$ defined by $\varPsi_I(t) = 0$ for $t \leqq a$, $\varPsi_I(t) = (t - a)/(b - a)$ for $a \leqq t \leqq b$, and $\varPsi_I(t) = 1$ for $t \geqq b$, is called the fundamental function associated with I. A set function K whose domain is $\mathscr{I}$ and whose range is in $L[X, Y]$ is said to be quasi-Gowurin if given a neighborhood V of θ_Y, there is a neighborhood U of θ_C such that $\sum \varPsi_{I_i} \cdot x_i \in U$ implies $\sum [K(I_i)](x_i) \in V$.

Finally, if $f \in C$ and σ is a partition of $[0, 1]$, then pf_σ denotes the polygonal function determined by σ and f.

2. The representation theorem.

Let C_θ denote the subspace of C such that $f(0) = \theta_X$.

THEOREM 2.1. *Suppose K is a set function on $\mathscr{I}$ with values*